UC San Diego: La Jolla CA, May 11, 2017

2017 Roger Dashen Memorial Lecture

Entanglement: Einstein's Gift to Quantum Mechanics

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- Einstein, quantum mechanics and "Entanglement"
- Novel entanglement properties of "Topological Quantum States of Matter" (Nobel Prize 2016)

- In public lectures about quantum mechanics, the usual subject is the 1927 **Heisenberg uncertainty principle:**

$$\Delta p \Delta x \geq \frac{1}{2}\hbar$$

Werner Heisenberg

"The precise position and velocity of a particle cannot be simultaneously measured"

or

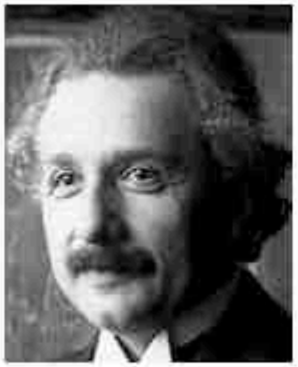
"One cannot measure a property of an object without affecting it"

I will not talk about this

There are (at least) two other key ingredients of quantum theory

- The (1925) Pauli exclusion principle for electrons (fermions)
- The "entanglement" property, first pointed out by Einstein Poldolsky and Rosen in 1935
  (all at I.A.S. Princeton)


Wolfgang Pauli

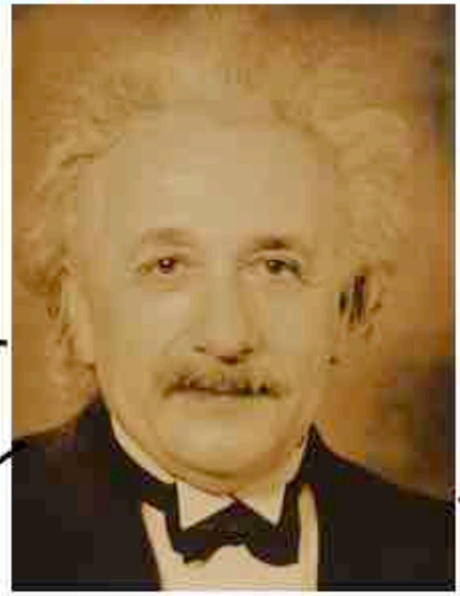

Albert Einstein


Boris Podolsky


Nathan Rosen

Relativity

Quantum Theory



Albert Einstein

(in Princeton, 1935)

- special relativity

1905

$$E = mc^2$$

- geometric theory of gravity

1915

$$G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} - \Lambda g_{\mu\nu}$$

"my biggest mistake"

cosmological constant = energy of empty space!

in fact, probably describes "dark energy"!

- photoelectric effect (shining light on metals)

1905 (Nobel prize 1921)

- Bose-Einstein condensation

1925 "cold atoms"

- Einstein-Podolsky-Rosen "paradox" (entanglement) 1935

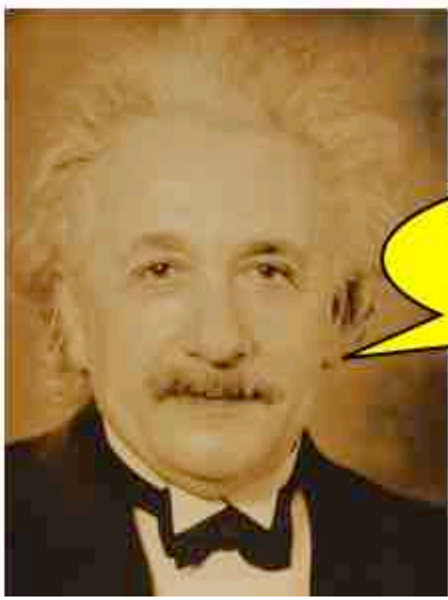
"second biggest mistake"(?),

just as brilliant as the first!

- Despite making an important early contribution (which, not Relativity, was cited for his Nobel prize!), by 1935 Einstein had rejected the quantum theory.....

- He identified a key property predicted by quantum theory which he felt just had to be wrong..

Erwin Schrödinger



"spooky action at a distance"*

*spukhafte Fernwirkung

*spukhafte Fernwirkung

just call it "Entanglement"

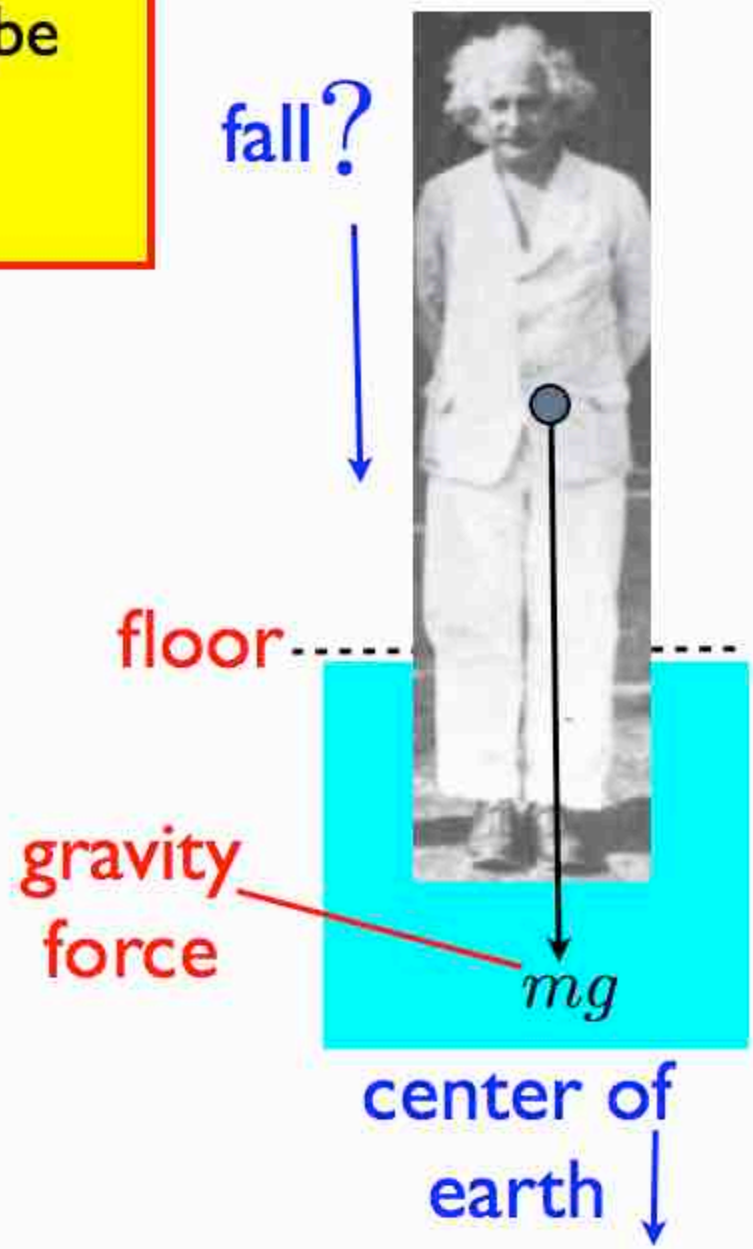
- “Quantum mechanics” has always been mysterious to the general public, more so than even Relativity
- It is also mysterious to Physicists, but as a “working quantum mechanic” I know it is the so-far always-confirmed “**true description”of the world** that dominates at small (atomic) lengthscales, while at human-sized scales, it is well approximated by earlier pictures, such as “classical” (Newtonian or Einsteinian) mechanics

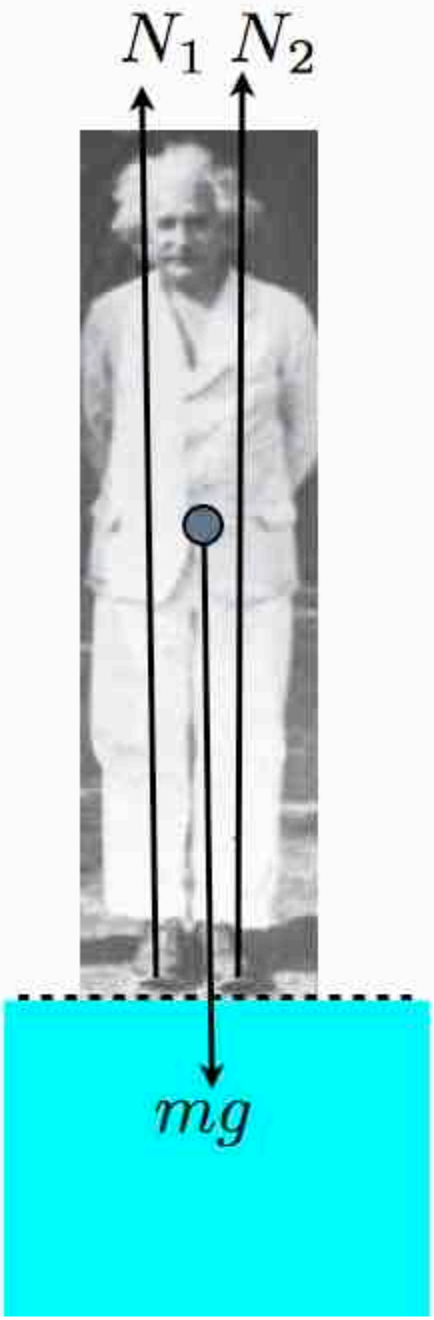
Originally, Einstein’s objections to quantum Entanglement seemed like a philosophical issue, like questions of the **compatibility of “free will” with quantum mechanics.** Today “Entanglement” is experimentally confirmed, and on its way to having technological importance for “quantum information processing”

- An important property of quantum mechanics is that two "fermion" particles such as electrons cannot be "in the same state" (such as in the same place)!

- Gravity is pulling us towards the center of the Earth, and the atoms out of which we are made are over 99.99% empty space..

- so why don't we fall through the floor? because of quantum mechanics!





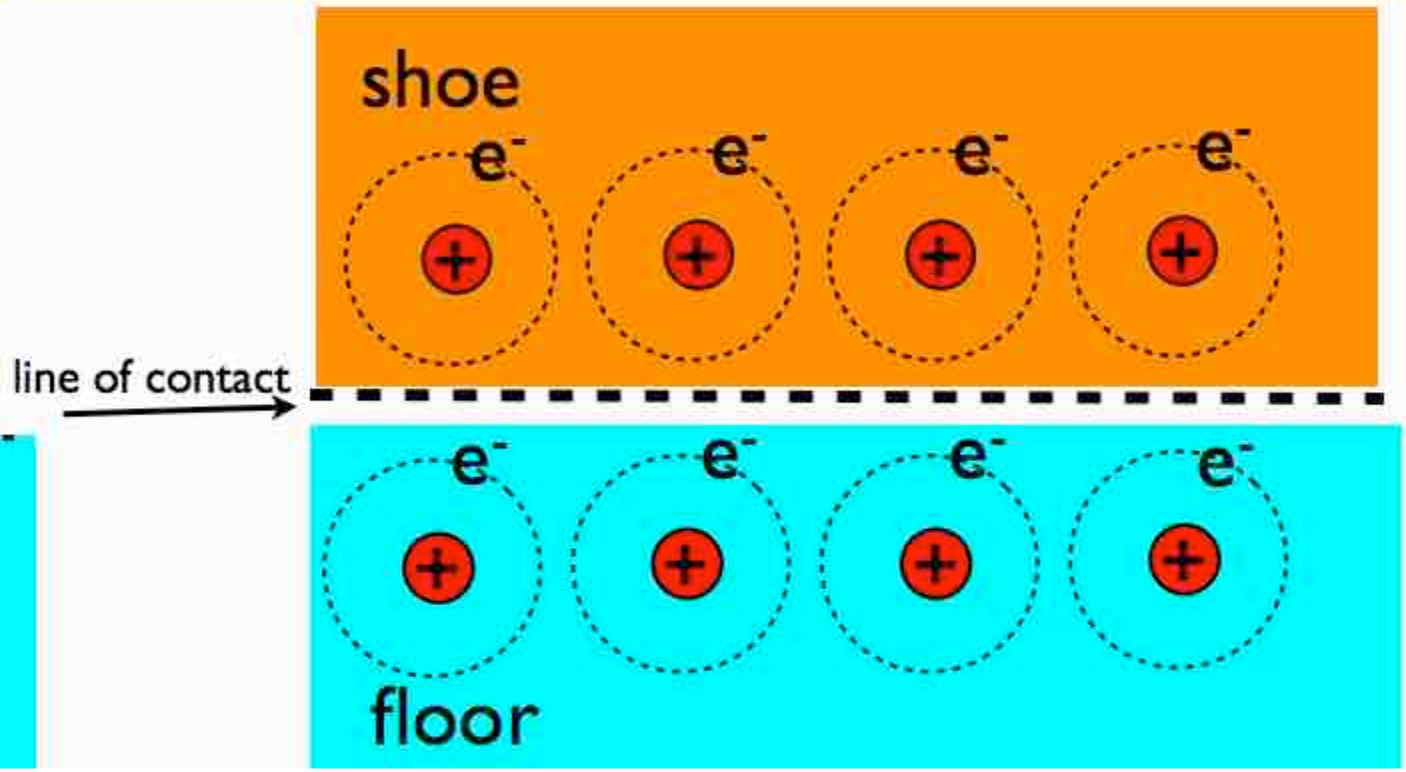
- Newton said that there was an upwards contact force or "normal force" exerted by the floor on the soles of our shoes that exactly balanced the downward force of gravity, so we don't fall though the floor

$N_1 + N_2 - mg = 0$ (net force vanishes, so we don't fall)

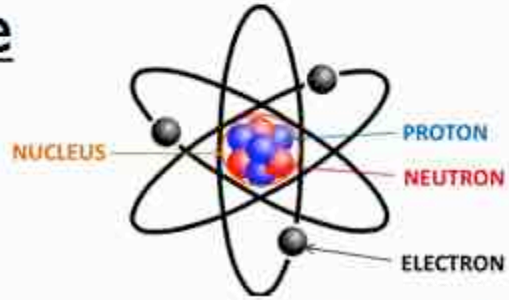
- Newton took the existence of the "normal force" between solid matter as an empirical fact that did not need explanation

- one of the fundamental properties of quantum mechanics is the Pauli exclusion principle, which says that two electrons ("fermions") cannot be in the same state

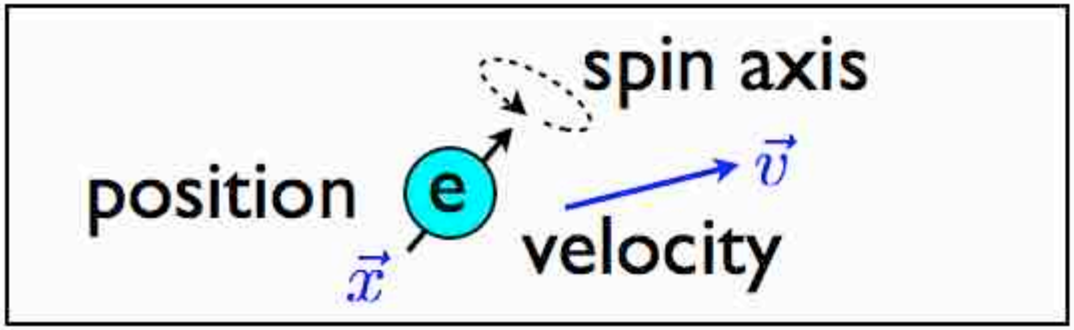
- electrons bound in the atoms locked together in the floor "exclude" the electrons bound in the atoms locked together in the soles of our shoes



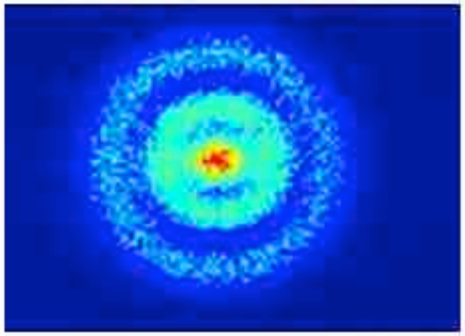
- One of the fundamental ingredients of the matter we are made of is the chemical bond, which does involve two electrons in the same place....
- This is possible because the "quantum state" that an electron is in does not just involve its position (and speed) but also an "internal" degree of freedom called its "spin" in analogy to a spinning top.

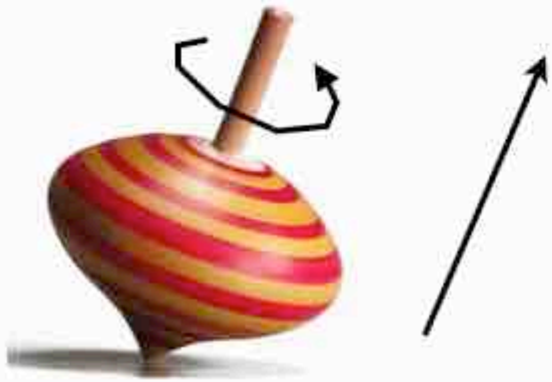


atom is a "box of electrons" bound to a nucleus



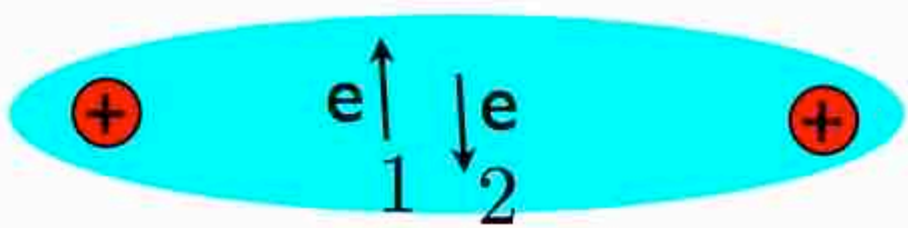
description of the state of an electron


real image of a hydrogen atom (one electron, in an excited state)



direction (axis) of
anticlockwise rotation (spin)

- The chemical bond is made of **two** electrons with exactly opposite spin, so they can occupy the same points in space



- whatever the direction the spin of electron 1 points in, the spin of electron 2 points in the other direction

- The chemical bond is the essential example of entanglement: whatever the direction that the spin of the first electron points in, it is opposite to that direction of the spin of the other electron

- but it can be pointing in <u>any</u> direction!

quantum states are represented by a "complex number" (amplitude)

$$\Psi = \frac{1}{\sqrt{2}} \quad (\uparrow\downarrow - \downarrow\uparrow) \quad = \frac{1}{\sqrt{2}}(\searrow\nwarrow - \nwarrow\searrow)$$

state is odd (changes sign) if the two electrons are swapped for each other

$$(\uparrow\downarrow - \downarrow\uparrow) = -(\downarrow\uparrow - \uparrow\downarrow)$$

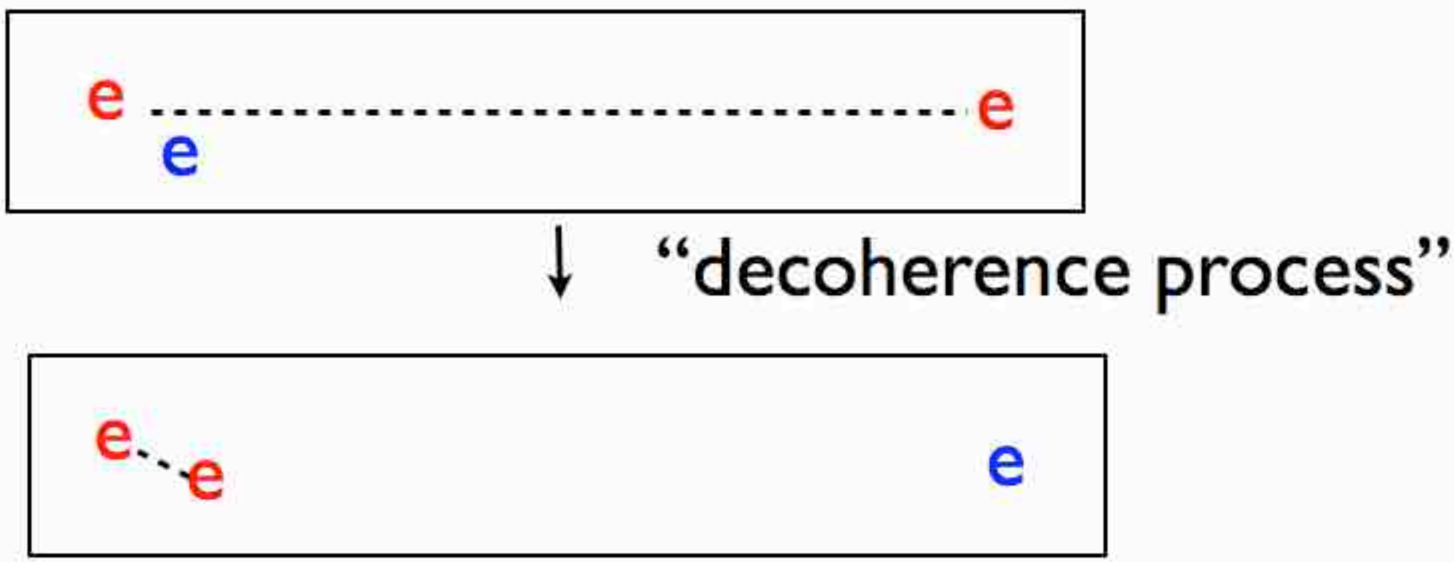
- length of chemical bond = one tenth of a nanometer
  (one billion nanometers = one meter)
- on this length scale, the two electrons in each chemical bond are completely entangled
- Einstein was worried about the theoretical possibility of states which were still entangled if the distance between the electrons was stretched to centimeters, meters, kilometers, or even light years.....

$$= \frac{1}{\sqrt{2}}\left(\uparrow \cdots\cdots\cdots\cdots \downarrow \quad - \quad \downarrow \cdots\cdots\cdots\cdots \uparrow \right)$$

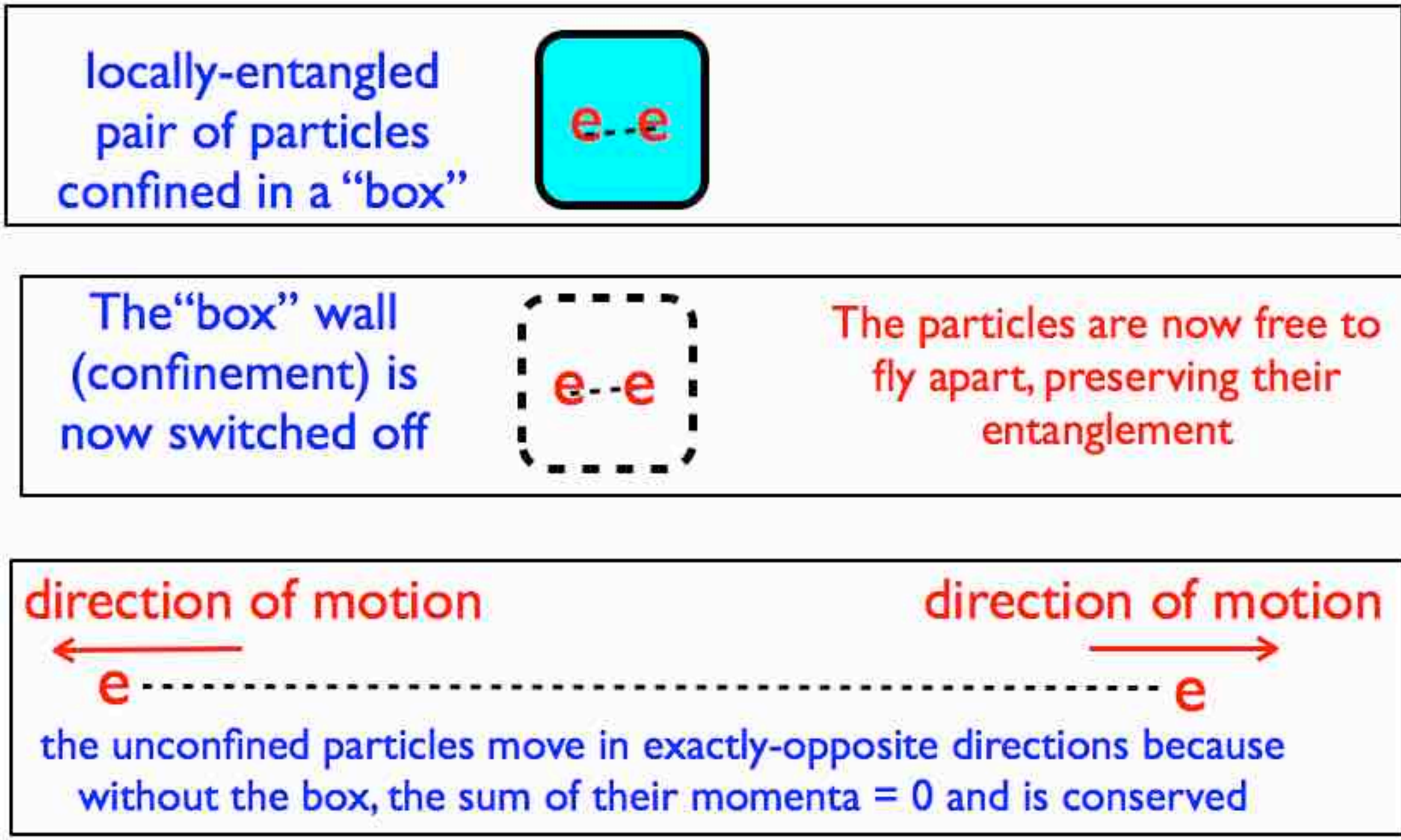
large distance

one meter is about 40 inches

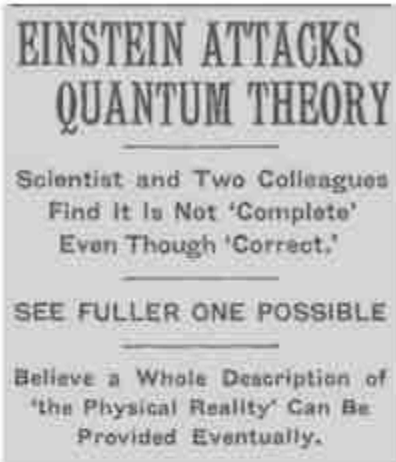
- When they are close together, the entangled state of the two electrons forming a chemical bond is protected by an "energy gap" that lowers their energy relative to other states. This is the chemical binding energy that makes the material stable.
- An entangled state of far-apart electrons is extremely fragile against interactions that entangle them with nearby electrons in their environment.....



- Einstein Podolsky and Rosen thought of the origin of what we now call an "EPR pair" of widely separated particles as the "decay" of an initially "confined state



New York Times May 4, 1935

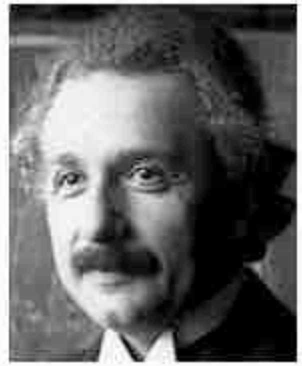

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

page one NYT headline!



Einstein

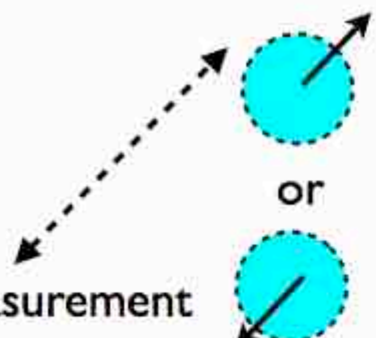
Boris Podolsky

Nathan Rosen

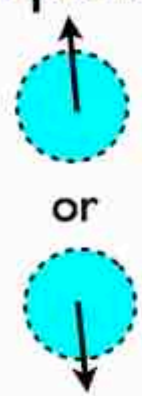
(all at I.A.S. Princeton)

- what worried Einstein was that until a measurement of the direction the spin of a particle was made, it could be pointing in any direction
- To make a measurement, a direction (axis) must be selected.
- The spin can then be found to point parallel or antiparallel to it

before measurement



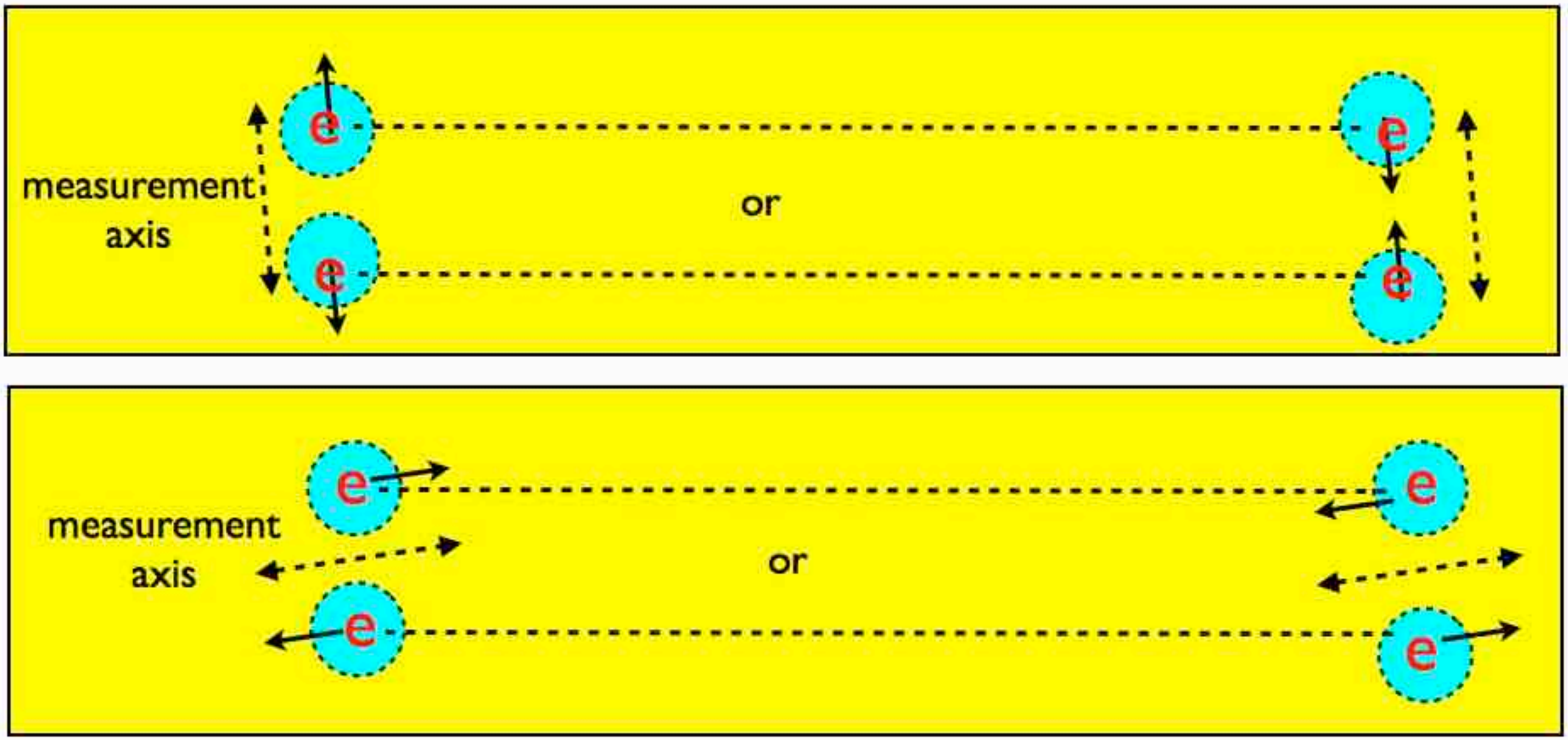
measurement axis



measurement axis

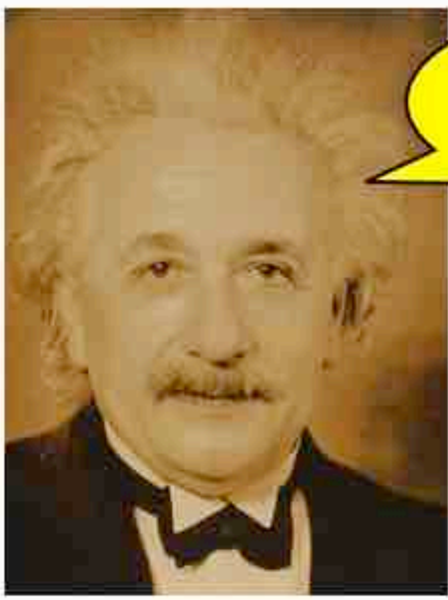
- An EPR pair flies apart, and the spin of each one is measured independently after they travel a long distance

- if both observers use the same measuring axis, they will ALWAYS get opposite results (IF DECOHERENCE HAS NOT OCCURED)

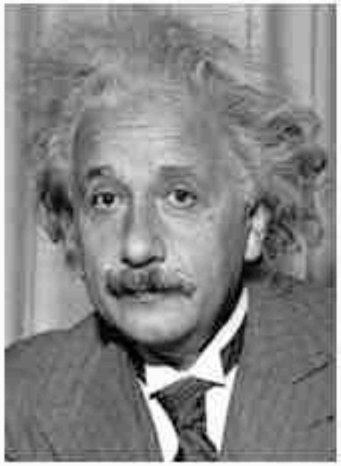


- Suppose the two observers agree to use one of two possible measurement axes, but only (independently) make their choices AFTER the pair has flown apart

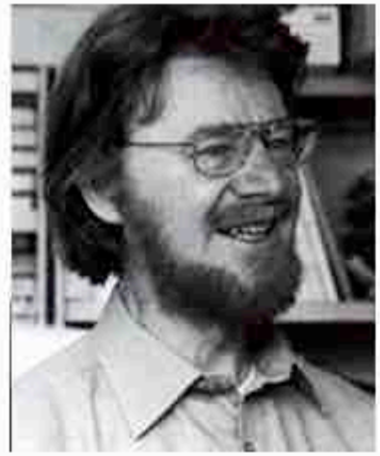
Making the choice of measurement axis is supposedly an exercise of "free will"



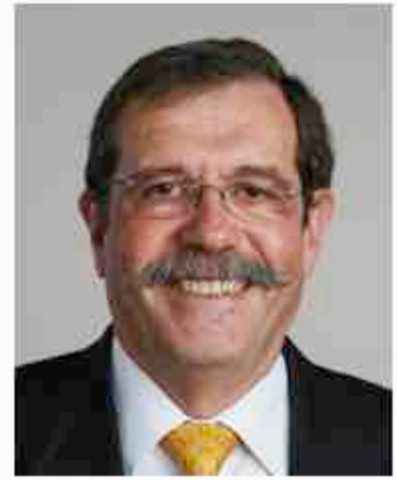
- was the result of the the experiment "preordained" at the moment the particles flew apart, or afterwards?



David Bohm



John Bell



Alain Aspect

- Einstein (incorrectly) though that if EPR could be tested experimentally, it would show the direction of each particles spin was already determined when the pair split, and show the quantum theory was not the true underlying description
- David Bohn and later John Bell made a deeper analysis, and (long after Einstein's death) in 1982 the French physicist Alain Aspect did the experiment and showed the quantum theory was 100% correct!

- Aspect's experiment used <u>photons</u> ("particles of light") where the two states of polarization of light are the analogs of the spin of the electron

linear polarization of light traveling towards you

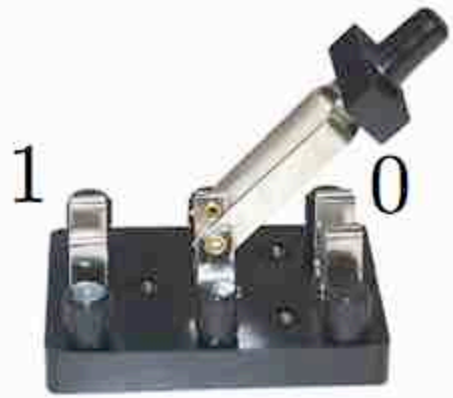
circular polarization of light traveling towards you

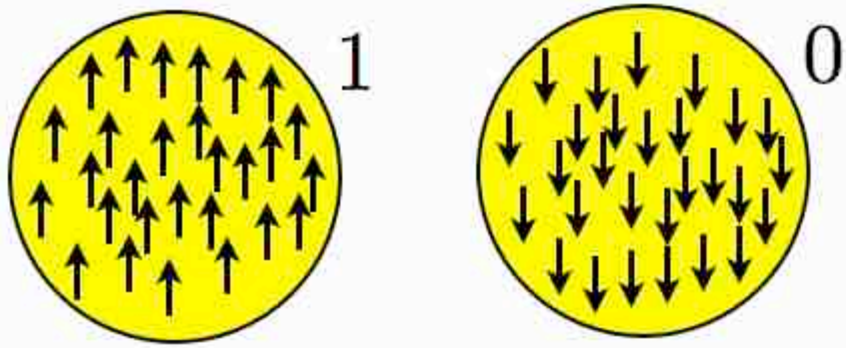
- using modern optical fibers, entangled EPR photon pairs separated by oceans can now be produced!

The era of discovering what "cool things" can be done with quantum mechanics, and gaining exquisite control of quantum states is only just beginning, 90 years after the laws of quantum mechanics were discovered

- The new ingredient is the the development of "quantum information theory" in partnership with deeper understanding and control of quantum states of matter in the 21st century

- Classical information is stored in a **"bit"** = a switch that is either "on" or "off" (1 or 0)

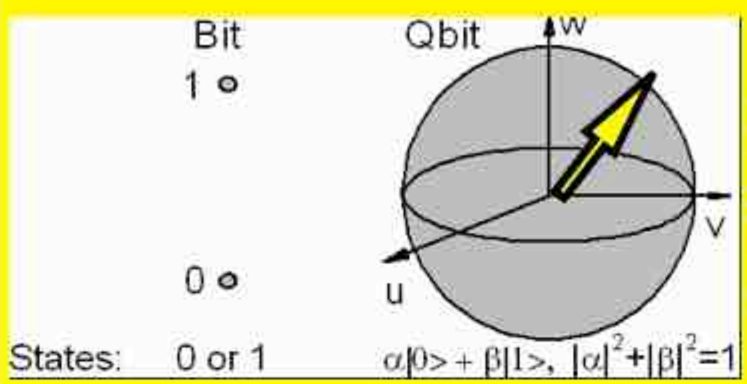




large numbers of spins, all "up" or all "down" behave "classically"

- quantum information is stored in the state of a single spin (a **"q-bit"**) which is read by a measurement with a choice of measurement axis.

any system with just two states is equivalent to an electron spin



spin can point in any direction on the "Bloch sphere"

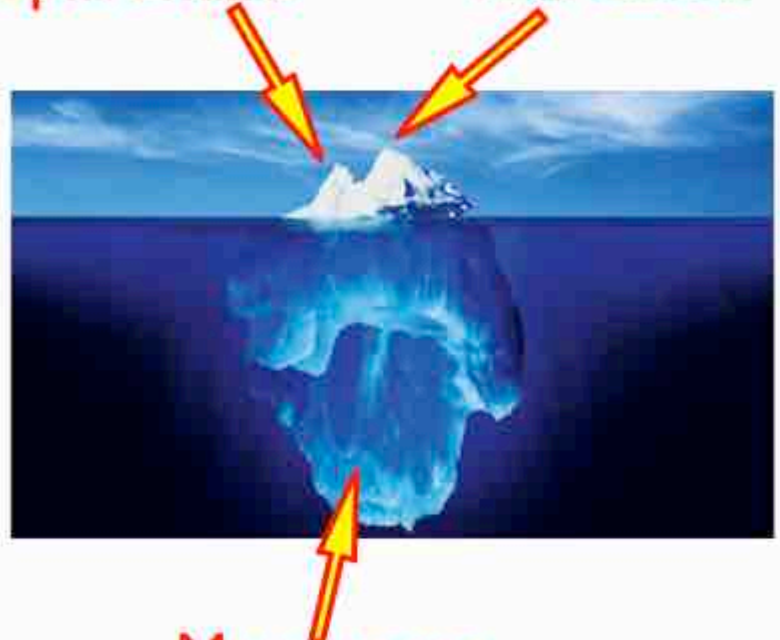
Topological quantum matter

- To the best of our knowledge, the basic laws of quantum mechanics have been known since the 1930's
- But just because we know them, it does not mean that we know all that they allow to happen!
- The laws of electromagnetism can be summarized in the four Maxwell equations, but it took a long time to understand all that can be done with them, and we are still learning!

The new developments are that we are starting to find some of the "cool things" that we had not guessed quantum mechanics can do!

- In 1980, "Condensed matter physicists" (who study solid and liquid matter, as opposed to elementary particles or atoms) thought they had a basic understanding of such systems.

- Independently, two examples of "weird properties" were found at about the same time, which we now know as a different kind of condensed matter in which <u>quantum entanglement plays a key role.</u>

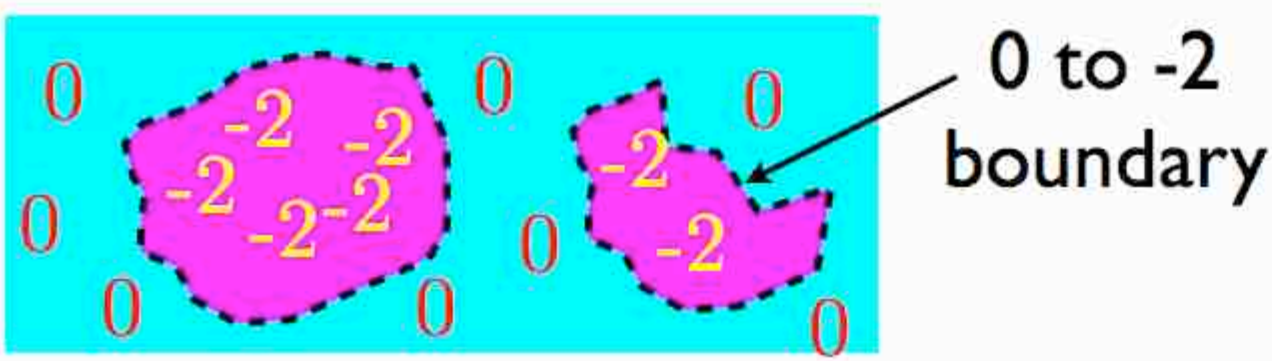


- "topological matter" differs from "ordinary matter" because it has (entanglement) properties that can be described by **whole numbers** (integers) like

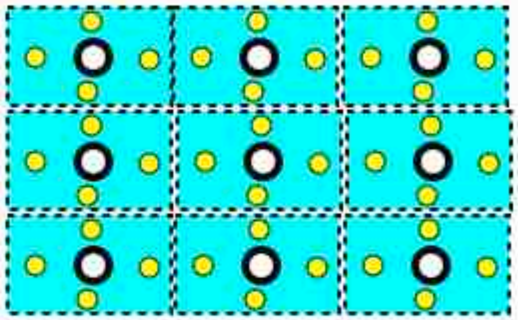
$$-1, 2, -5, 4, ...$$

- "ordinary matter" is described by "boring" numbers like 0 or 1.

- states of matter described by different whole numbers cannot smoothly change into one another, and there is always something interesting at the boundary between different types.

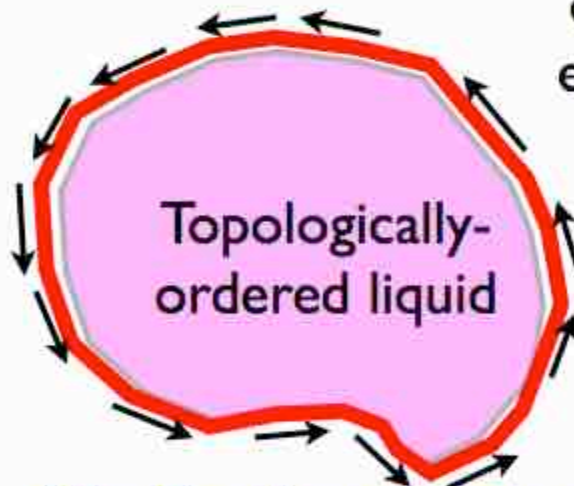


Regular versus "topological" condensed matter



conventional solid order

quantum Hall
electron liquid



Broken symmetry
inside the
condensed matter

- Nothing special at edges

No Broken symmetry
inside the condensed matter

so what is different?

- Unusual excitations inevitable present at boundary between "topological" and "regular" matter

- "Topological states" of quantum matter generally have some unusual property associated with "quantum entanglement" that forces there to be some inevitable feature at the edge of the material where the entanglement stops

- The topological classes of matter are generally classified by whole numbers (integers) which cannot change continuously and take "trivial" values such as 0 or 1 in "regular" matter.

- This makes their properties immune to small amounts of disorder or "dirt" in the material, in contrast to "usual" material properties which may require ultra-clean systems (e.g. silicon wafers). The disorder must exceed some threshhold to cause an abrupt change from "topologically non-trivial" to "trivial" properties.

- The classic example in mathematics is the classification of closed surface by the "genus" or number of holes

- a shape such a sphere with 0 holes can easily be distorted by small forces, but only a violent event can change it into a doughnut (torus) with 1 hole

- There is no such thing as "0.3604 ± 0.0005 holes"!

- **geometric** properties (such as curvature) are ***local*** properties
- but integrals over local geometric properties may characterize global topology!

Carl Friedrich Gauss

Gauss-Bonnet (for a closed surface)

$$\int d^2\boldsymbol{r}(\text{Gaussian curvature}) = 4\pi(1 - \text{genus})$$

$$= 2\pi(\text{Euler characteristic})$$

product of principal radii of curvature $\frac{1}{R_1 R_2}$

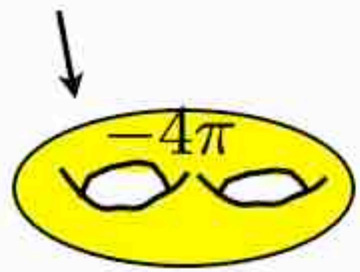
$$4\pi r^2 \times \frac{1}{r^2} = 4\pi(1-0)$$

- trivially true for a sphere, but non-trivially true for any compact 2D manifold

Santa Lucia Bun?

Ball

Bagel



“Swedish Pretzel”

German Pretzel

mug

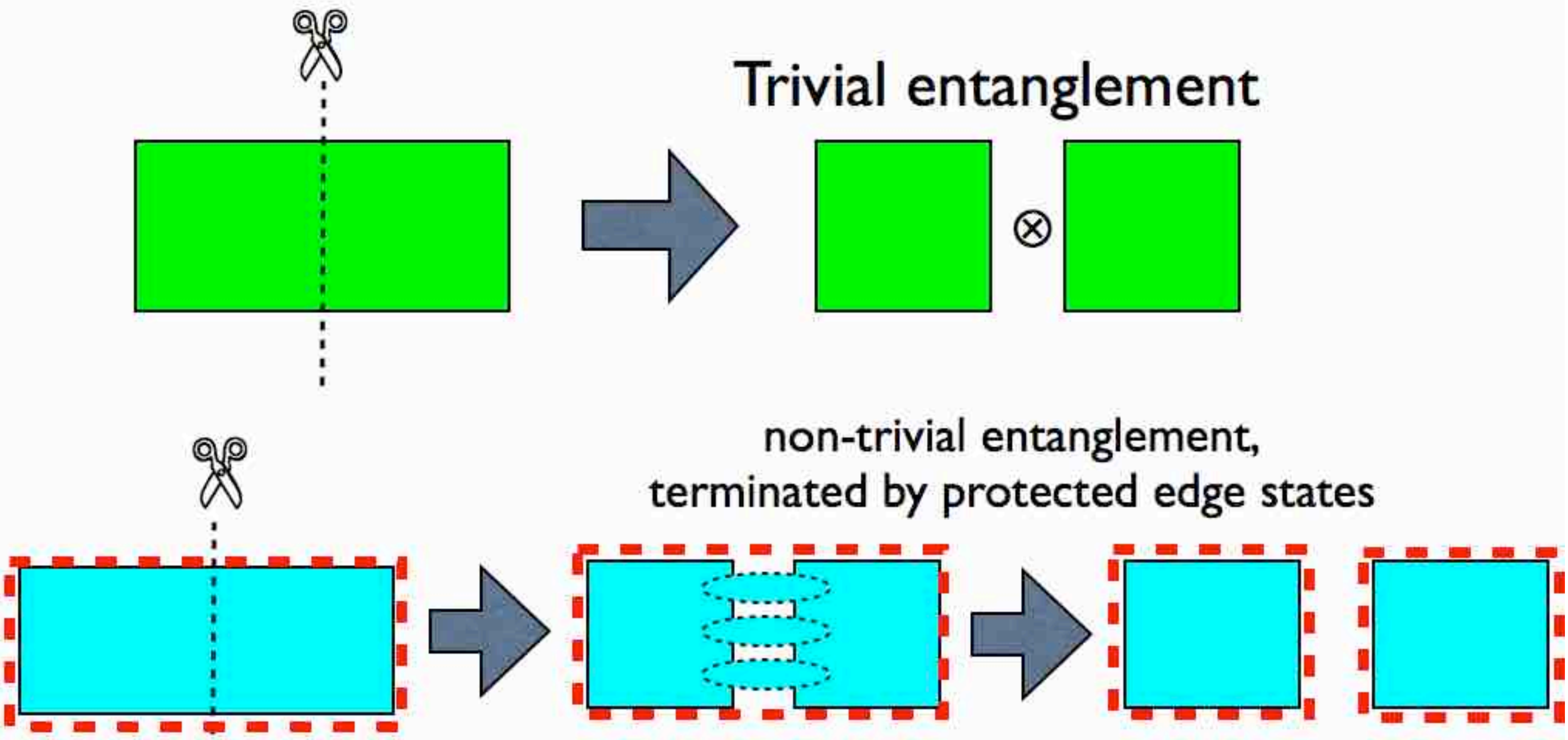
coffee cup

“loving cup”

???

- Before 1980 “topological quantum matter” was unknown
- Around that time some states of matter with surprising and unexpected properties started to be found
- It took some quite time for it to be understood that these systems had unusual properties because they were topologically different from “regular” states of matter
- We are finally achieving a unified understanding of this

- In recent years, it has been realized that quantum condensed matter can exhibit unexpected properties associated with long range <u>quantum entanglement</u>



- Topologically-trivial states of insulating matter can in principal be assembled by bringing their constituent atoms together, with all electrons remaining bound during the process
- Topologically non-trivial states of matter cannot be adiabatically connected to atomic matter. At some point during their formation, bound electrons are liberated, then rebound in a state with non-trivial entanglement

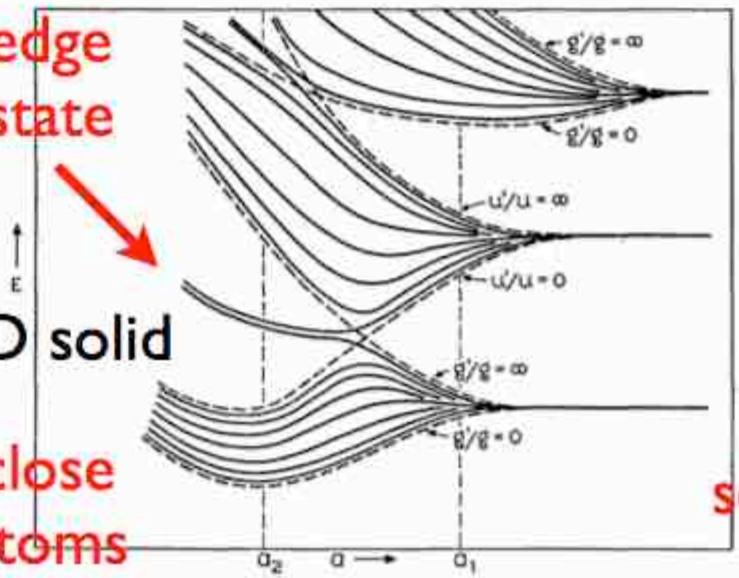
- One of the striking characteristic properties of band topological insulators (or "Symmetry-Protected Topological States") is their edge states

Shockley 1939

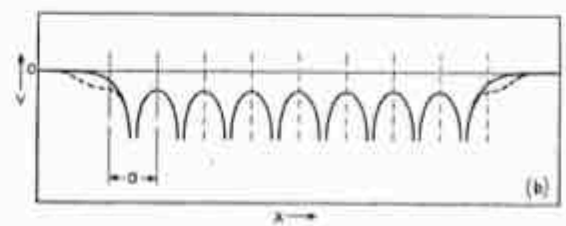
edge state

1D solid

close atoms



widely separated atoms

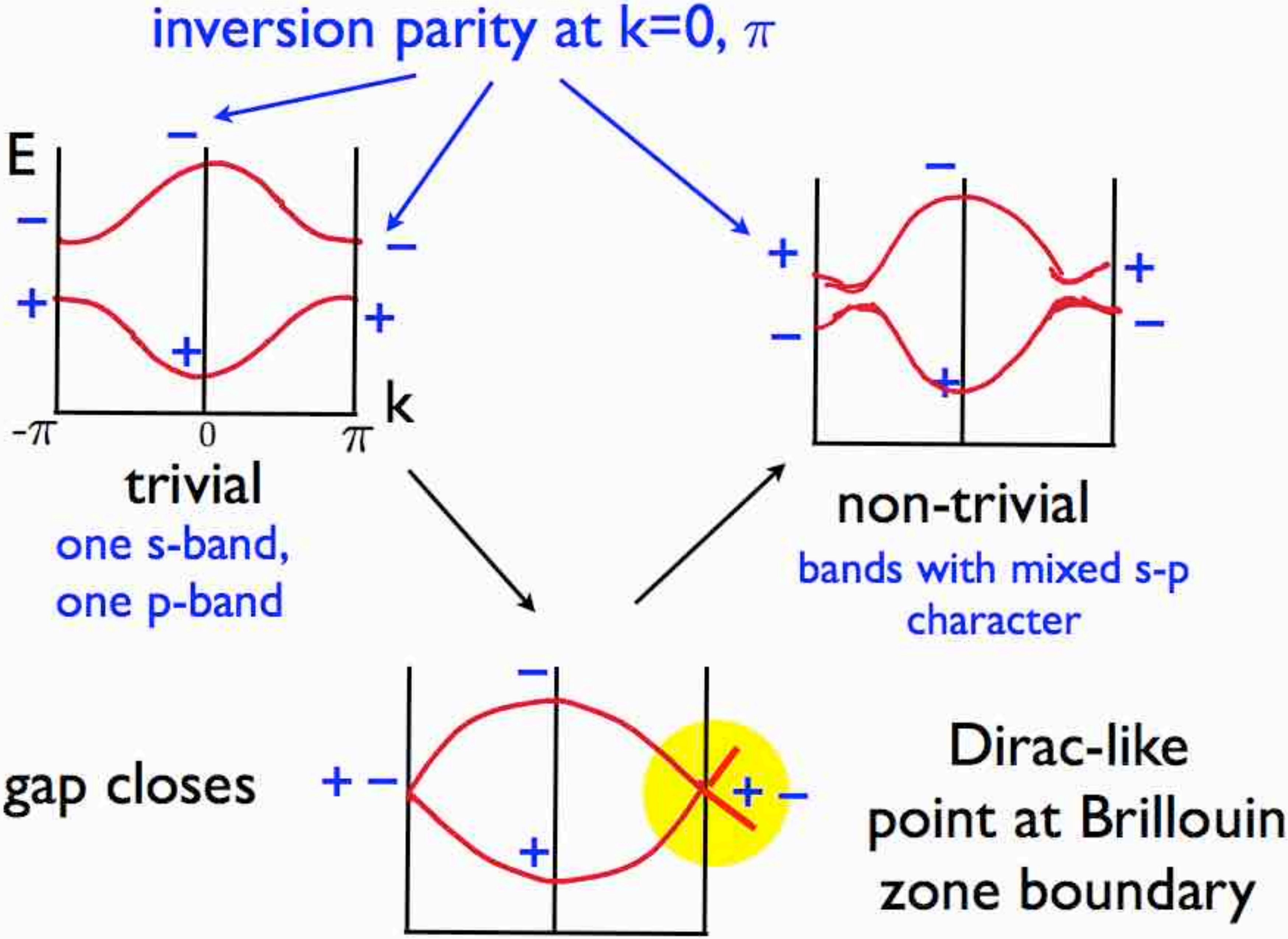


Fermi level pinned to edge state if neutral charge +1/2 electron if full , -1/2 if empty, per edge

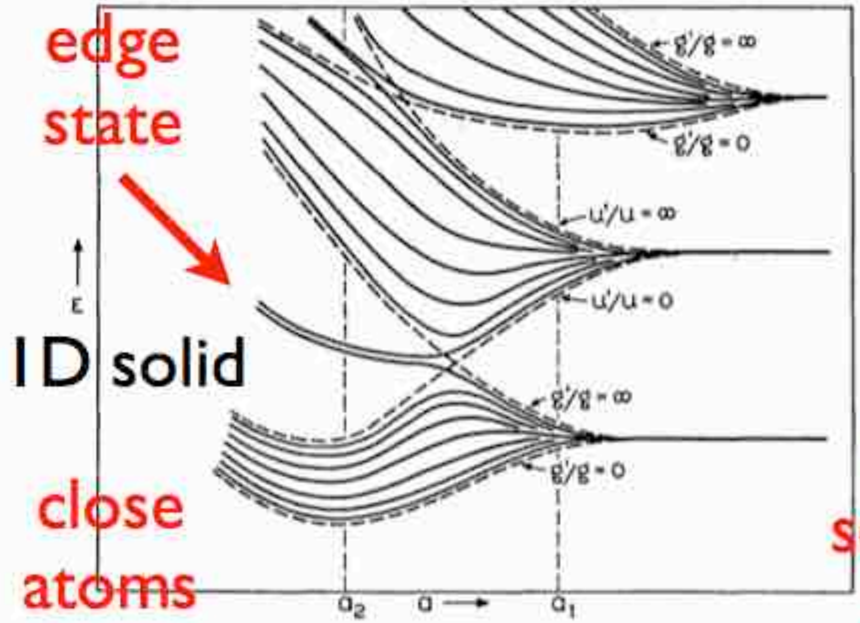
protective symmetry: spatial inversion

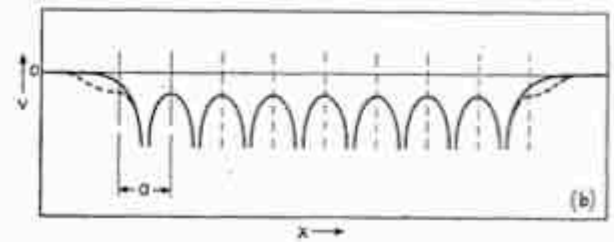
Z_2 invariant: $I_{k=0} \times I_{k=\pi} = \pm 1$

s-p band-inversion at k = π



inversion parity at k=0, π
E
k
-π
0
π
trivial
one s-band,
one p-band
non-trivial
bands with mixed s-p
character
gap closes
Dirac-like
point at Brillouin
zone boundary

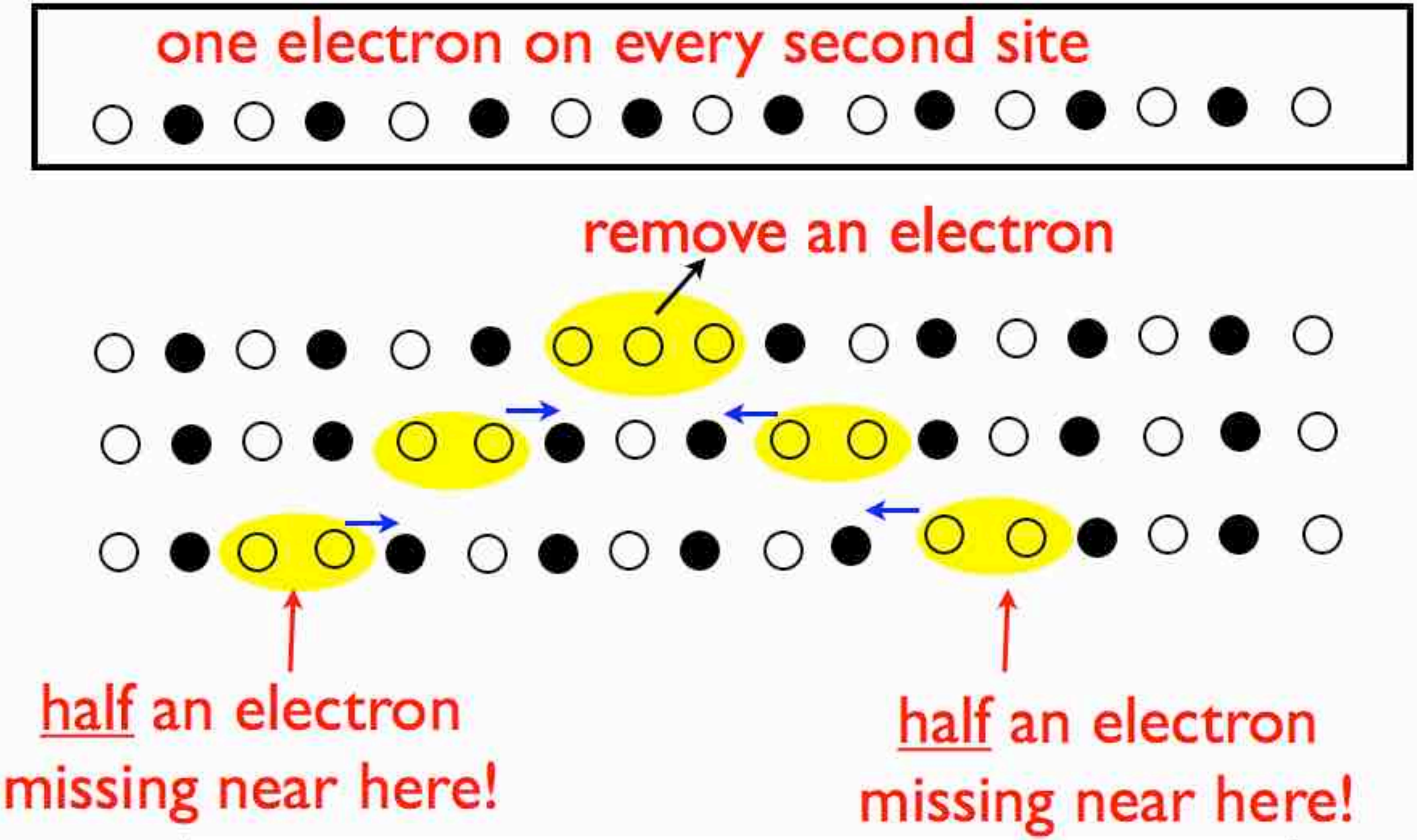




Fermi level pinned to edge state if neutral charge +1/2 electron if full , -1/2 if empty, per edge

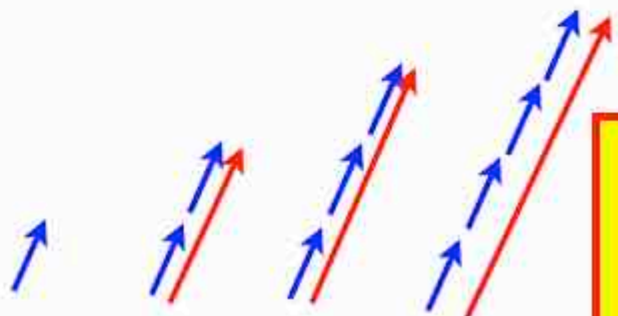
- If both edge states are occupied, there is **<u>one</u>** extra electron, 50% at one edge, 50% at the other (half an electron at each edge)
- If both are empty there is half a hole at each edge

- Fractionalization of the electron, in an interacting "density-wave state" on a lattice where repulsive Coulomb forces keep electrons apart



unexpected fractionalisation is typical in topological states!

- In heavier atoms, electrons in a group of atomic orbitals can become "locked together" so they are always parallel, and give rise to magnetism



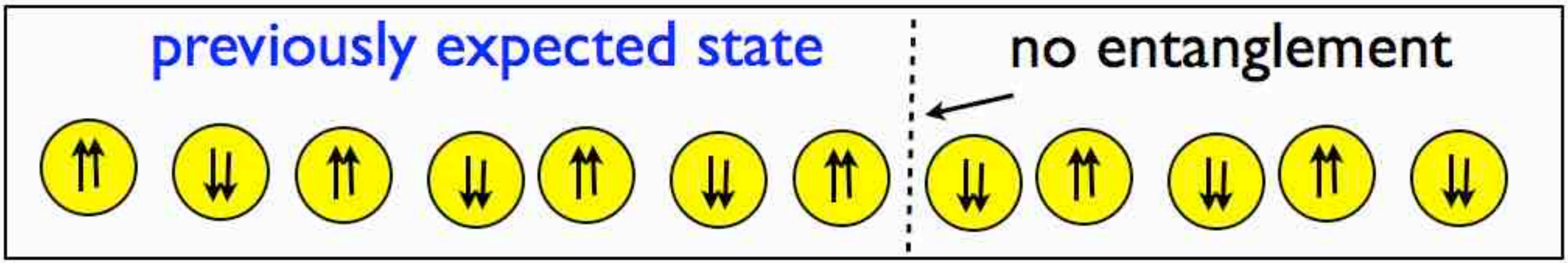
The total spin S of the complete group is either an integer or half an integer

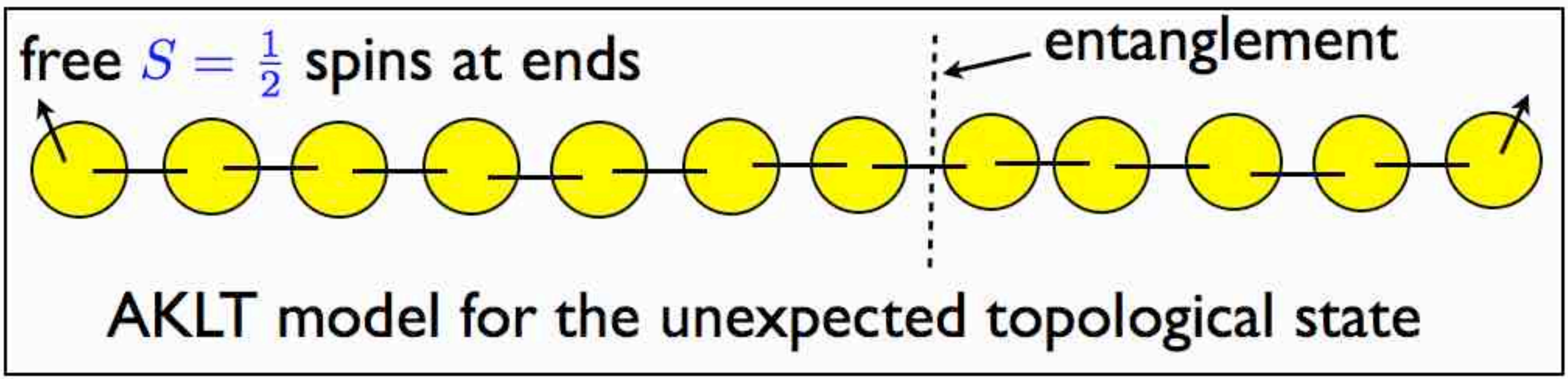
$$S = 2S \times \tfrac{1}{2}$$

electron spin

a whole number (even or odd)

- In 1981 I discovered that a S=1 chain on spins could have a novel state that is now understood as the simplest of the topological states

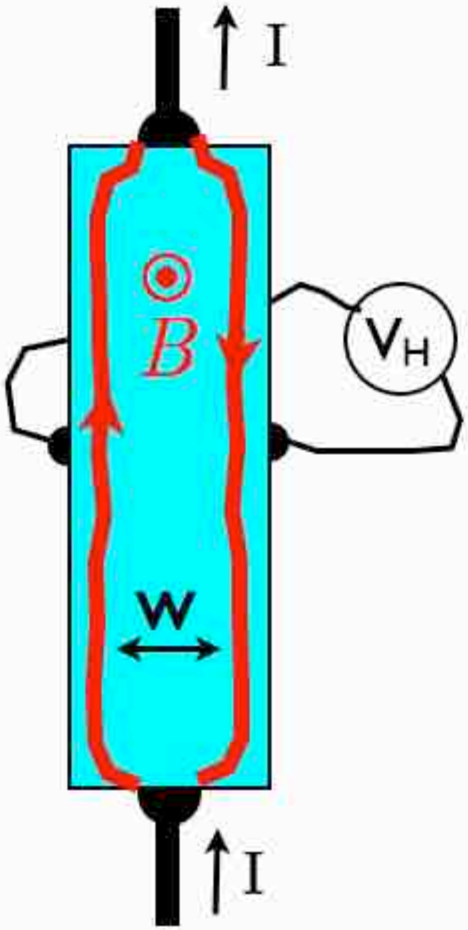




- I found that magnetic chains where S was an integer behaved completely differently from those with half-integer S, challenging a picture that had been accepted for over 40 years
- This was extremely controversial, and was rejected for publication for a while, until experiments showed it was true!
- We now know that the unexpected behavior was because the S=1 system is a topological state..

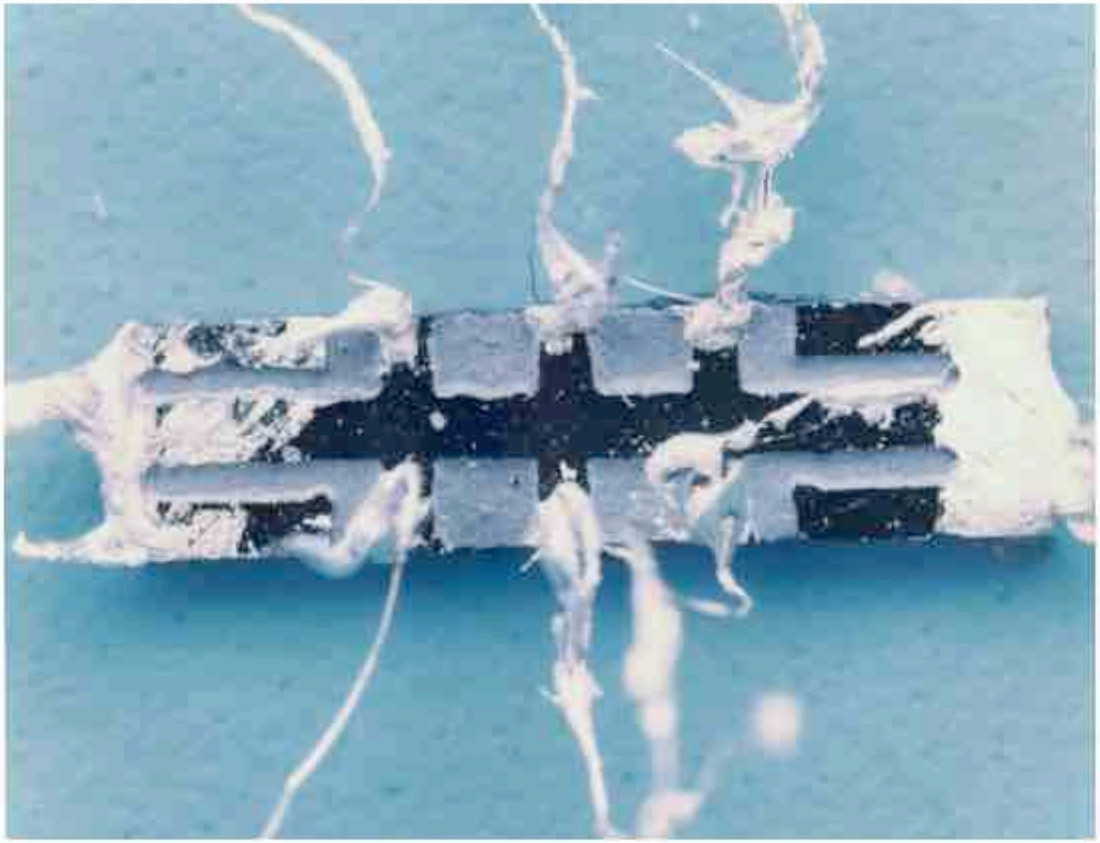

$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
Kungliga Svenska
Vetenskapsakademien har
den 4 oktober 2016
beslutat att med det
NOBELPRIS
som detta år tillerkännes den
som inom fysikens område gjort
den viktigaste upptäckten
eller uppfinningen gemensamt belöna
F. Duncan M. Haldane
J. Michael Kosterlitz och David J. Thouless
för teoretiska upptäckter av topologiska
fasövergångar och
topologiska materiefaser.
STOCKHOLM DEN 10 DECEMBER 2016

- Surprise #2: the quantum Hall effect



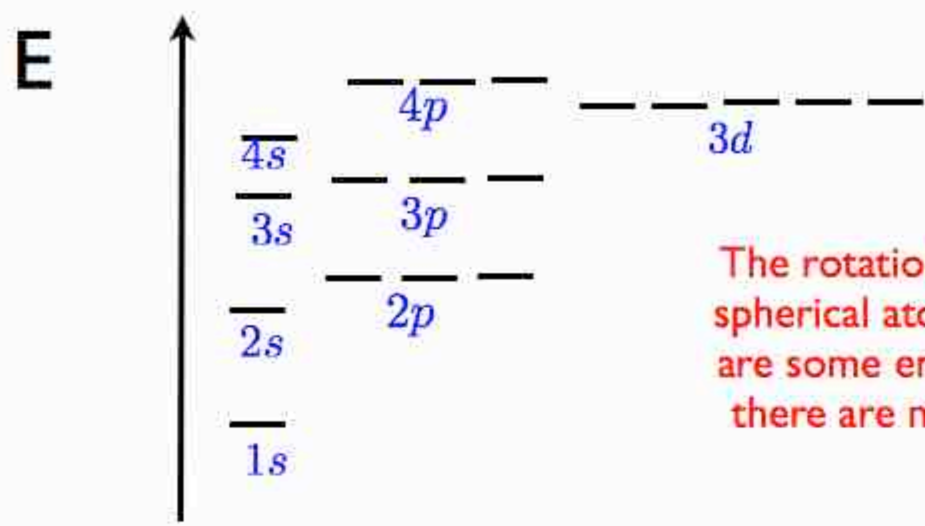
$$I = \sigma_H V_H \qquad \sigma_H = \frac{e^2}{2\pi\hbar} \times \frac{p}{q}$$

- why was this so accurately quantized at low T in a non-uniform sample ?
- initially people looked in vain for relativistic corrections, etc.
- It turned out that the reason is topological, and the only corrections are from tunneling between the left and right edges of the sample, exponentially small for large w.



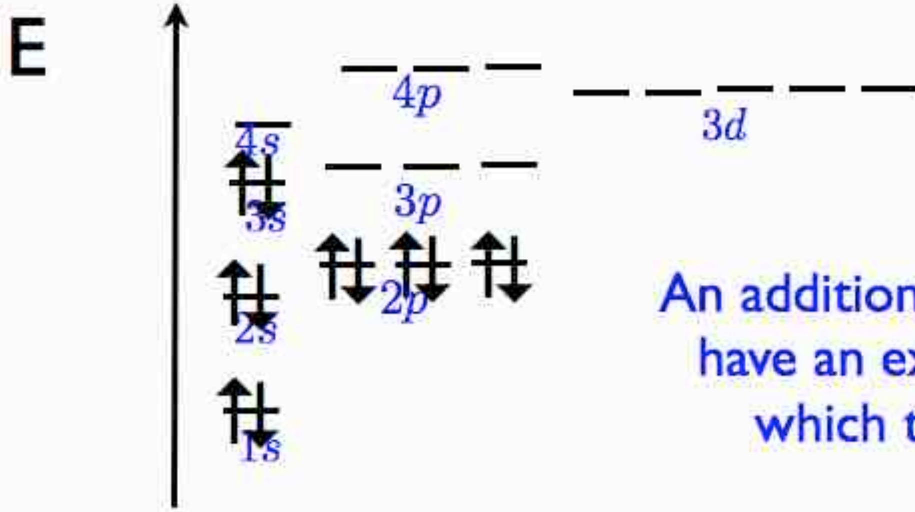
- The device (made by Gossard) in which Störmer and Tsui found the fractional quantum Hall effect (1982)

- In high school chemistry, we learn that electrons bound to the nucleus of an atom move in closed orbits around the nucleus , and quantum mechanics then fixes their energies to only be one of a <u>discrete set of energy levels.</u>



The rotational symmetry of the spherical atom means that there are some energy levels at which there are more than one state

- This picture (which follows from the **Heisenberg uncertainty principle**) is completed by the **Pauli exclusion principle**, which says that no two electrons can be in the same state or "orbital"

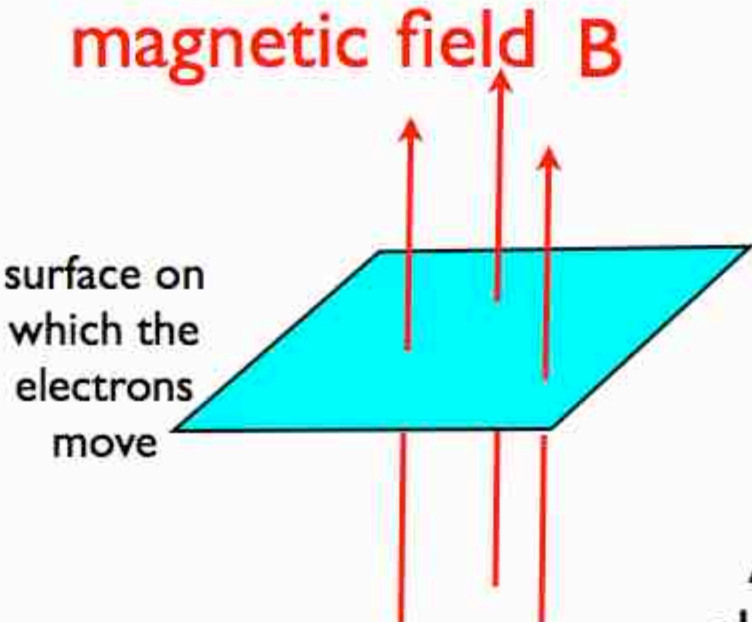


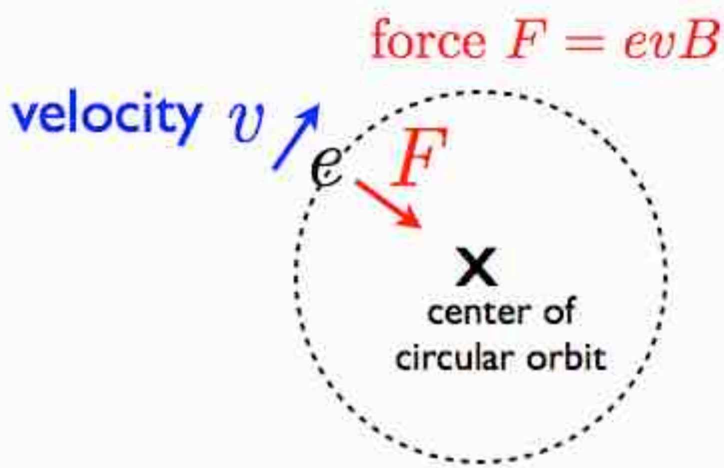
Occupied orbitals of the Calcium atom (12 electrons)

An additional ingredient is that electrons have an extra parameter called "spin" which takes values "up" (↑) and "down" (↓)

This allows two electrons (one ↑, one↓) to occupy each orbital

- If electrons which are not bound to atoms are free to move on a two-dimensional surface, with a magnetic field normal to the surface, they also move in circular orbits because there a magnetic force at right angles to the direction in which they move
- In high magnetic fields, all electrons have spin $\uparrow$ pointing in the direction of the magnetic field

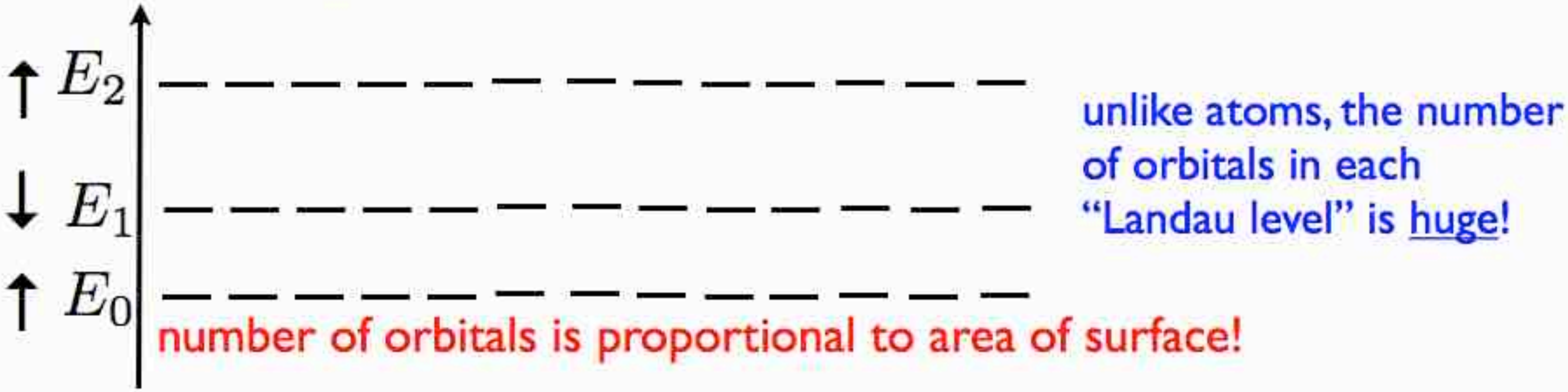




As in atoms, the (kinetic) energy of the electron can only take one of a finite set of values, and now determines the radius of the orbit (larger radius = larger kinetic energy)

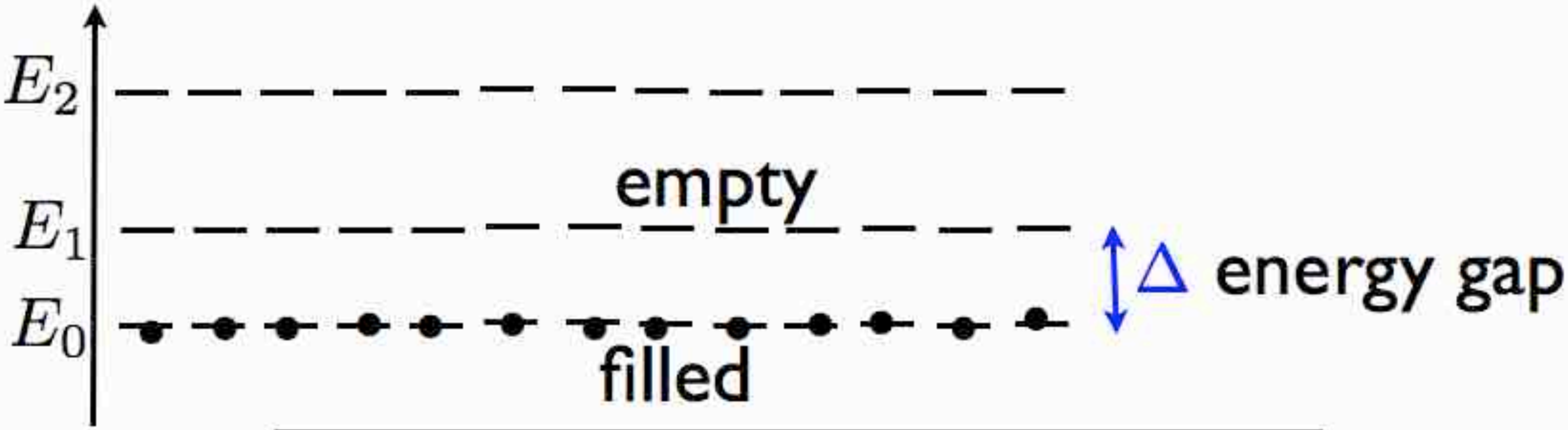
- As with atoms, we can draw an energy-level diagram:

(spin direction is fixed in each level)



degeneracy of Landau level = $\dfrac{\text{Total magnetic flux through surface}}{\text{(London) quantum of magnetic flux}}$ = $\dfrac{B \times \text{area}}{h/e}$

- For a fixed density of electrons let's choose the magnetic field B just right, so the lowest level is filled:



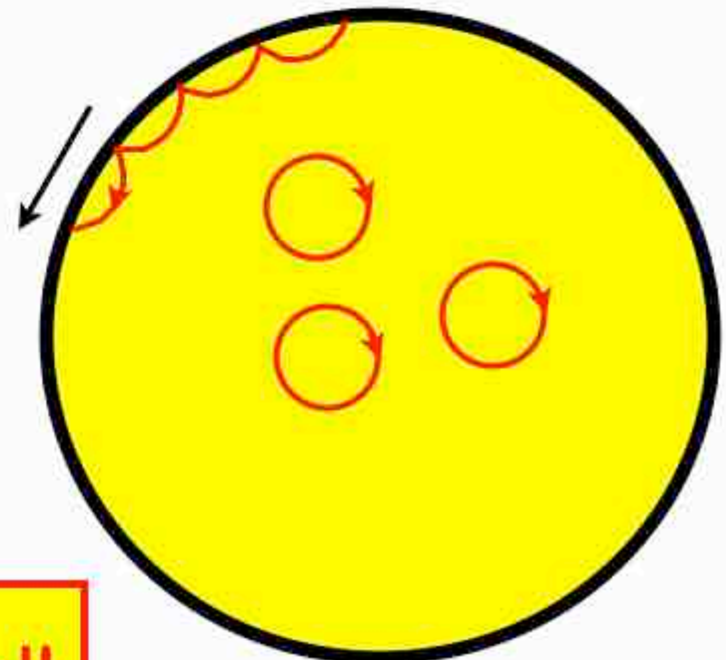
electron density if if n Landau levels are exactly filled	$\frac{eB}{2\pi\hbar}$	← independent of details

- This appears to describe the integer quantum Hall states discovered by Klaus von Klitzing (Nobel Laureate 1985)

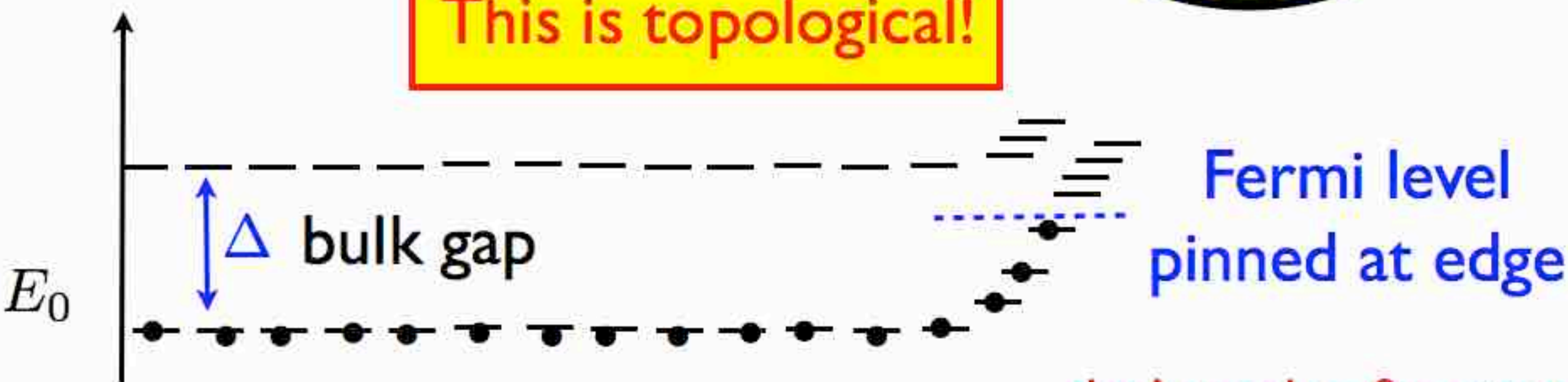
- BUT: seems to need the magnetic field to be "fine-tuned".

- In fact, this is a "topological state" with extra physics at edges of the system that fix this problem

- counter-propagating "one-way" edge states (Halperin)
- confined system with edge must have edge states!



This is topological!

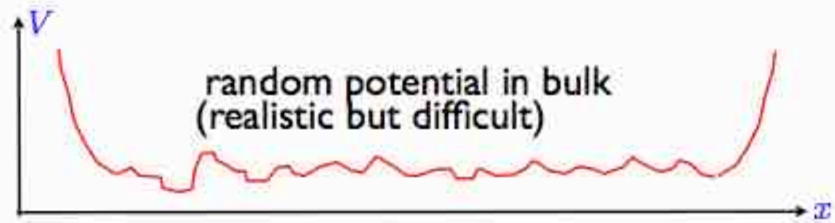


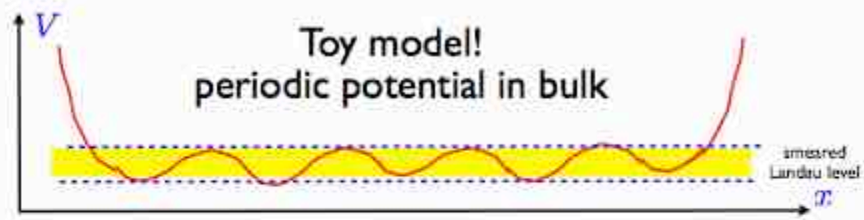
don't need to fine-tune magnetic field

- Until the work of David Thouless (with Kohmoto,Nightingale and den Nijs ,TKNN) the role of topology had not been recognized:

David Thouless

- Up till then, work on understanding the (integer) quantum Hall effect had focussed on the role of the inhomogeneity of the substrate:





- TKNN had the brilliant idea to replace the complicated "realistic" model with a much more tractable "toy model", a periodic lattice

- TKNN found a Kubo formula that forced the Hall conductance in units of e^2/h to be an integer

- The mathematical physicist Barry Simon was intrigued by their formula, and using the just-discovered Berry phase, recognized it as Chern's extension of the topological Gauss-Bonnet formula to more abstract curvature on more abstract two-dimensional compact manifolds, in this case the 2D Brillouin zone.

$$\mathcal{F}_n^{ab}(\boldsymbol{k}) = \frac{1}{2i}\int d^d\boldsymbol{r} \left(\frac{\partial u_n^*}{\partial_{k_a}} \frac{\partial u_n}{\partial_{k_b}} - \frac{\partial u_n^*}{\partial_{k_a}} \frac{\partial u_n}{\partial_{k_b}} \right)$$

unit cell

Berry curvature

an antisymmetric tensor in momentum space

- The two-dimensional 1982 TKNN formula

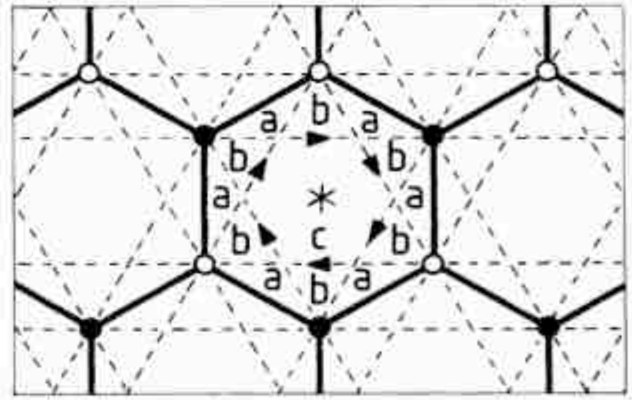
$$\sigma_H^{ab} = \frac{e^2}{\hbar} \sum_n \int_{\mathrm{BZ}} \frac{d^2\boldsymbol{k}}{(2\pi)^2} \mathcal{F}_n^{ab}(\boldsymbol{k})$$

This is an integral over a "doughnut": the torus define by a complete electronic band in 2D

- Though in retrospect it is clear, it was not recognized until 1988 that the TKNN result was not restricted to “exotic” lattice systems in high magnetic fields, but also applied to a model I introduced for a “zero field (“anomalous”) quantum Hall effect,

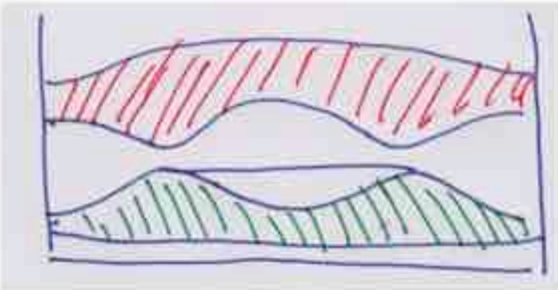
The 2D Chern insulator

- This was a model for a "quantum Hall effect without Landau levels" (FDMH 1988), now variously known as the "quantum anomalous Hall effect" or "Chern insulator".

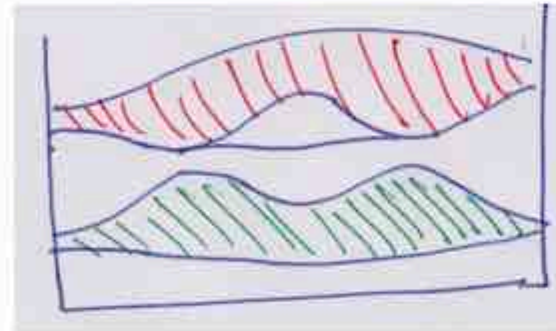


- It just involves particles hopping on a lattice (that looks like graphene) with some complex phases that break time reversal symmetry.

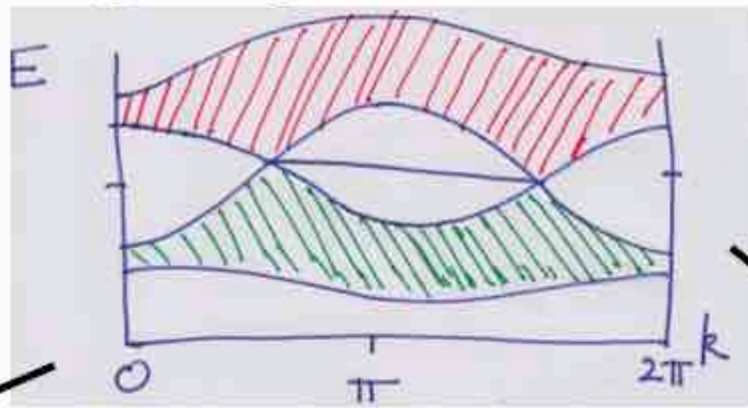
- By removing the Landau level ingredient, replacing it with a more standard crystalline model the "topological insulators" were born

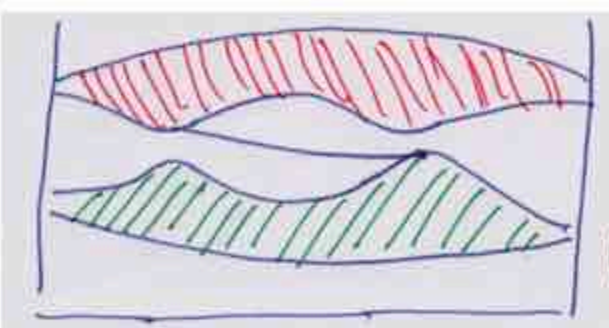


Broken inversion

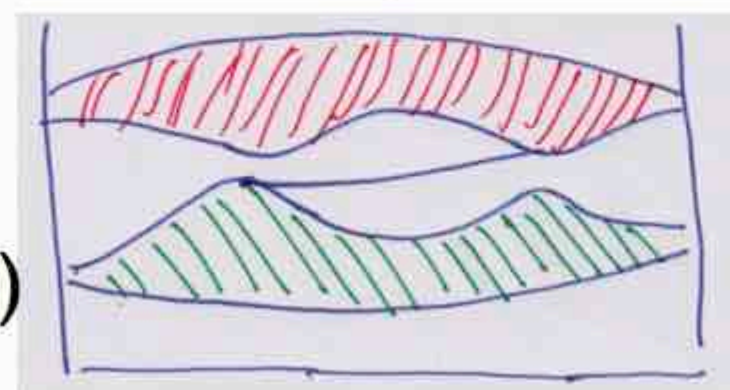


- gapless graphene "zig-zag" edge modes





Broken time-reversal (Chern insulator)



Analogs of quantum Hall edge states in photonic crystals

Haldane and Raghu, Phys. Rev. Lett.100, 013904 (2008)

- Predicted theoretically that using magnetooptic (time-reversal-breaking) materials, photonic analogs of electronic quantum Hall systems could be created where topologically-protected edge modes allow light to only travel along edges in one direction, with no possibility of backscattering at obstacles!

- Effect was experimentally confirmed recently at MIT (Wang et al., Nature 461, 775 (8 October 2009).

- Obvious potential for technological applications! (one-way loss-free waveguides)

microwaves go around obstacle!

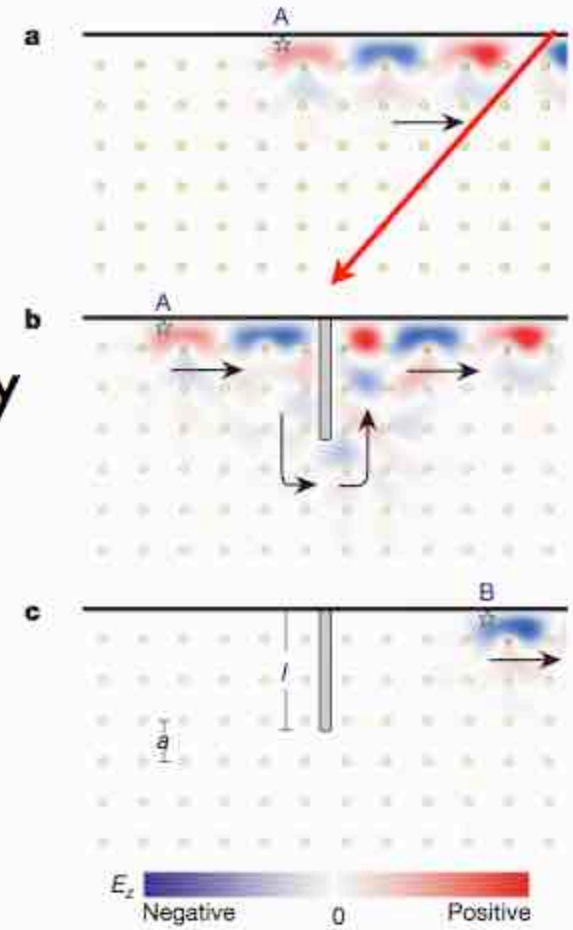
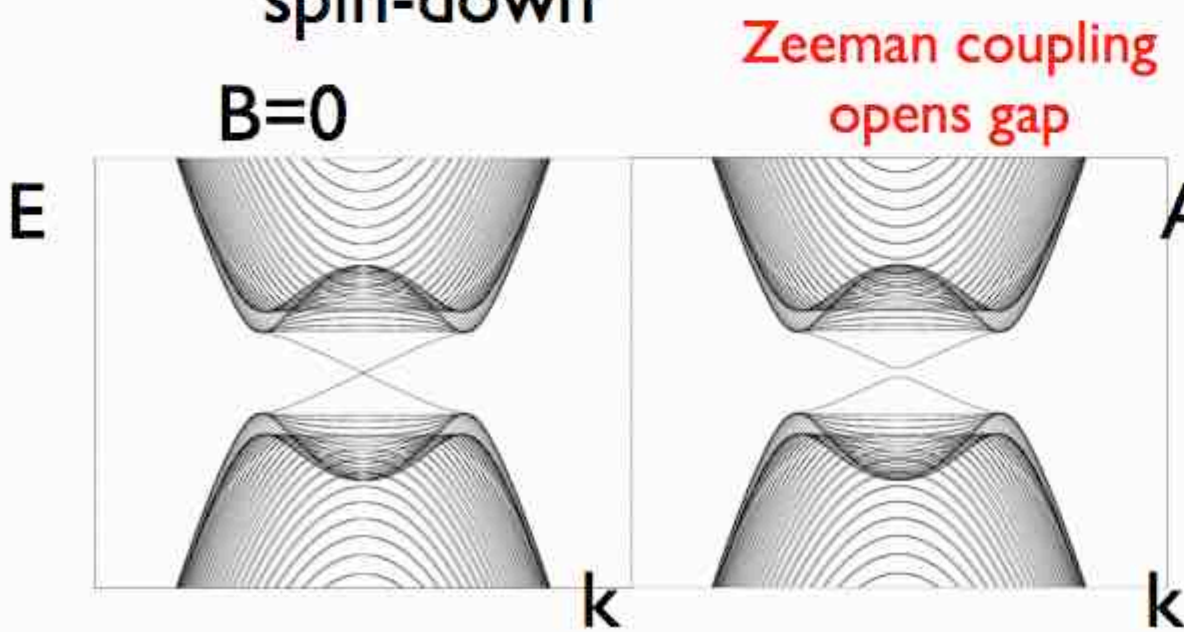


Figure 2 | Photonic CESs and effects of a large scatterer. a, CES field distribution (E_z) at 4.5 GHz in the absence of the scatterer, calculated from finite-element steady-state analysis (COMSOL Multiphysics). The feed

(from Wang et. al)

Kane and Mele 2005

- Two conjugate copies of the 1988 spinless graphene model, one for spin-up, other for spin-down



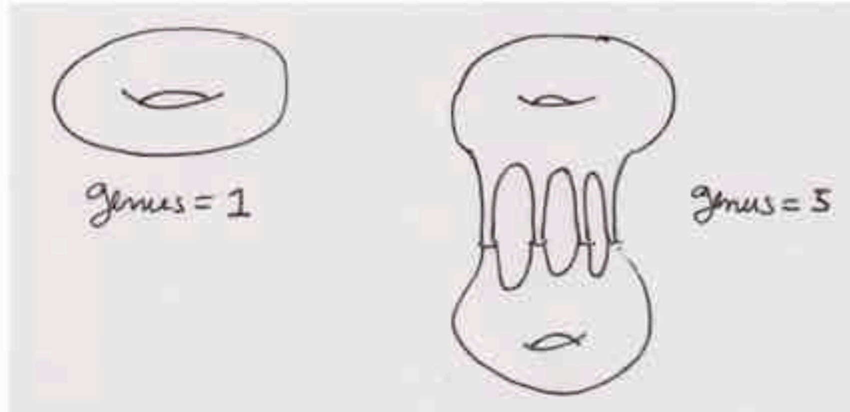
At edge, spin-up moves one way, spin-down the other way

If the 2D plane is a plane of mirror symmetry, spin-orbit coupling preserves the two kind of spin. Occupied spin-up band has chern number +1, occupied spin-down band has chern-number -1.

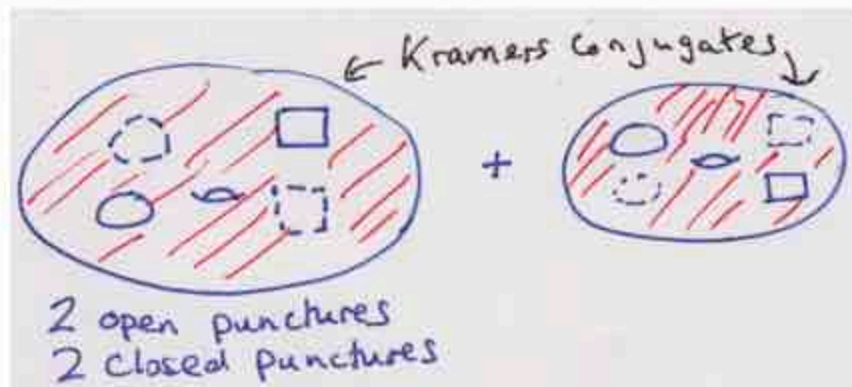
An explicitly gauge-invariant rederivation of the Z2 invariant

FDMH unpub.

- If inversion symmetry is absent, 2D bands with SOC split except at the four points where the Bloch vector is 1/2 x a reciprocal vector. The generic single genus-1 band becomes a pair of bands joined to form a genus-5 manifold



- This manifold can be cut into two Kramers conjugate parts, each is a torus with two pairs of matched punctures. In each pair, one puncture boundary is open one is closed.

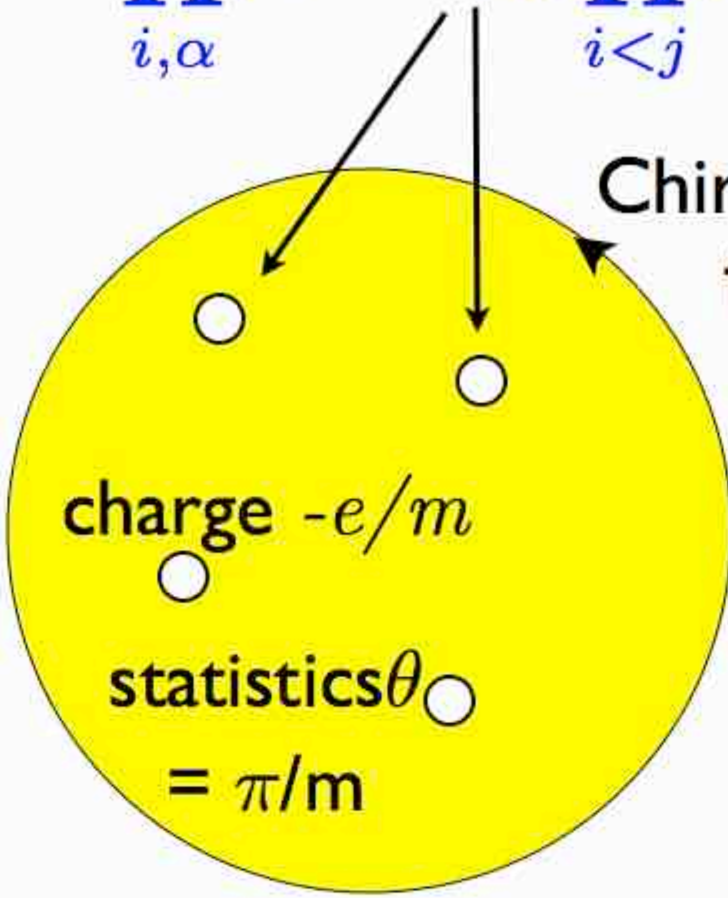


- At around the the same time (1982), another remarkable type of 2D “topological matter” emerged: first the integer (von Kilitzing), then the fractional(Stormer, Tsui, Gossard; Laughlin) quantum Hall effect.
- These also showed robust edge states which (as time-reversal symmetry is broken by a high magnetic field) travel one way around an edge.

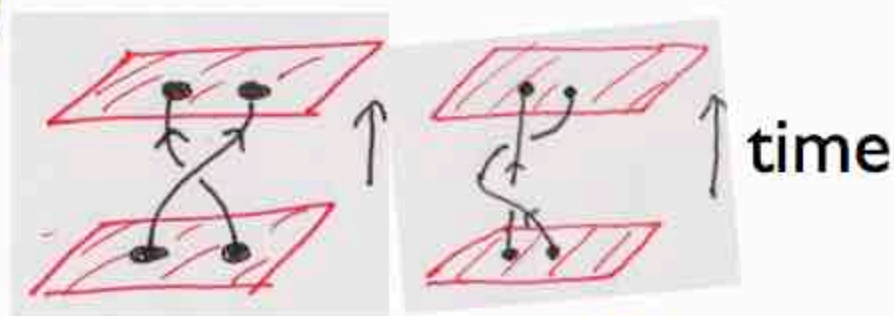
fractional-charge, fractional statistics vortices

$$\Psi = \prod_{i,\alpha}(z_i - w_\alpha)\prod_{i<j}(z_i - z_j)^m \prod_i e^{-\frac{1}{4\ell_B^2} z_i^* z_i}$$

Chiral edge states at edge of finite droplet of fluid (Halperin, Wen)

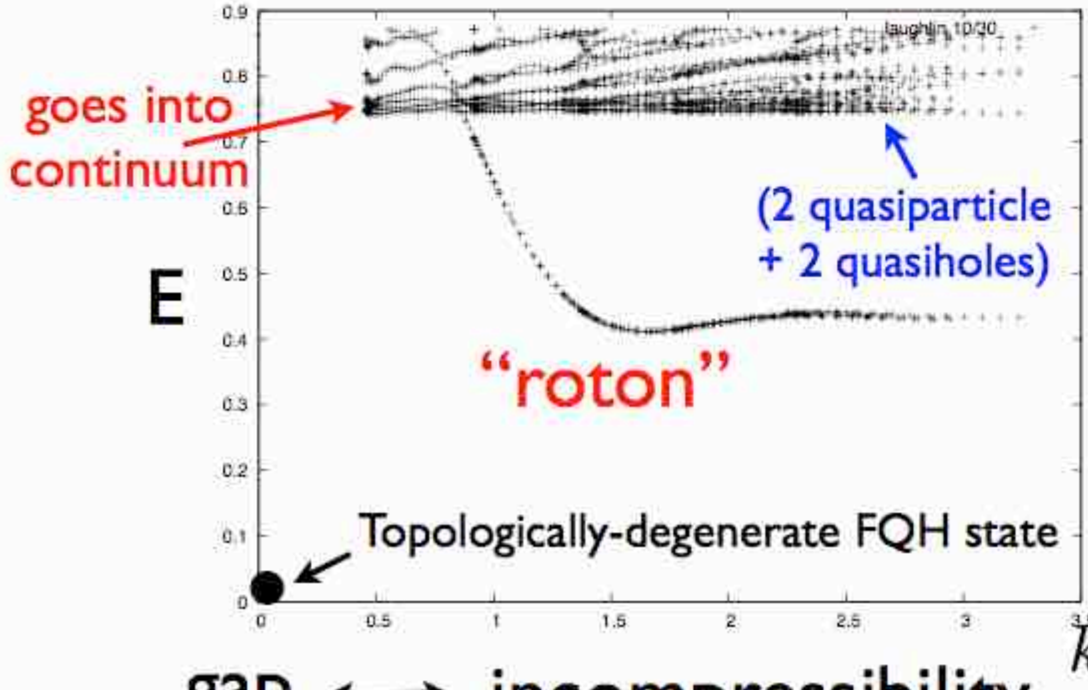


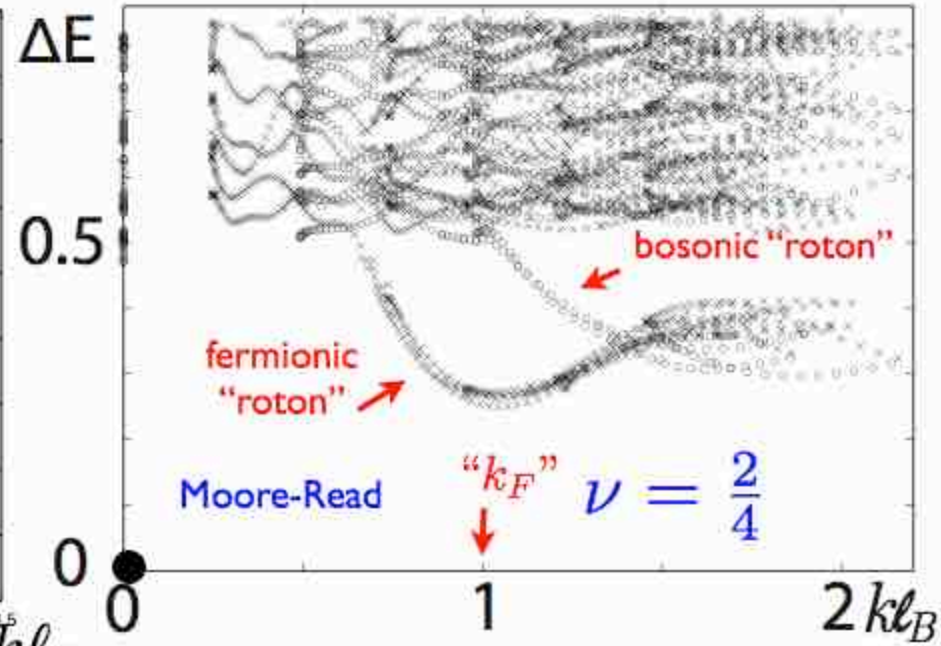
e.g., m=3



$e^{i\theta}$ $e^{2i\theta}$

- Even more remarkable than "Abelian" quantum Hall states are the "Non-Abelian" ones pioneered by Moore and Read that can hide "quantum information" in their entanglement structure.





gap $\longleftrightarrow$ incompressibility

Collective mode with short-range V_1 pseudopotential, 1/3 filling (Laughlin state is exact ground state in that case)

Collective mode with short-range three-body pseudopotential, 1/2 filling (Moore-Read state is exact ground state in that case)

- momentum $\hbar$k of a quasiparticle-quasihole pair is proportional to its **electric dipole moment $\mathbf{p_e}$**

$$\hbar k_a = \epsilon_{ab} B p_e^b$$

gap for electric dipole excitations is a MUCH stronger condition than charge gap: fluid **does not transmit pressure through bulk!**

- The "topological q-bit" that Alexei Kitaev wants to use..

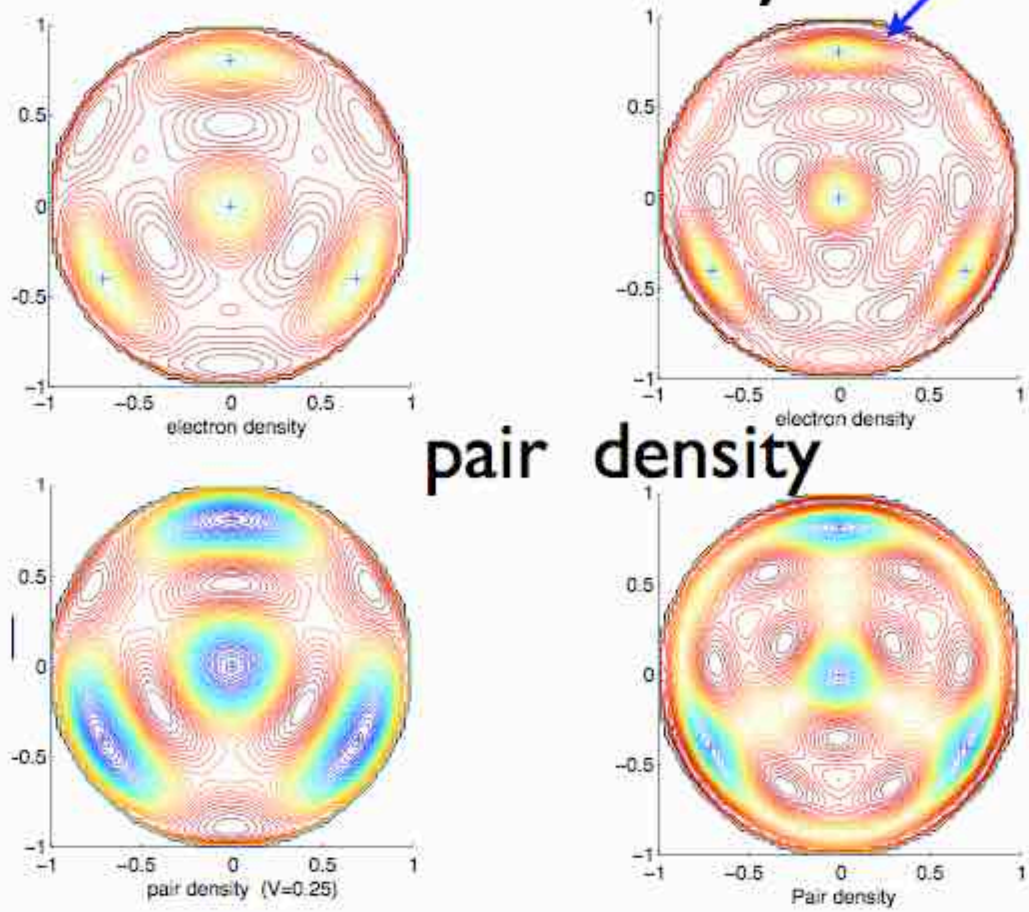
- 4 Majorana zero modes, fixed local fermion parity
- Leaves a "hidden" two-fold degeneracy that can be used as a q-bit to store/process information

Because the distance between quasiparticles (vortices) is finite, states are slightly distinguishable!

(exponentially-small tunneling effect)

charge-1/4

electron density



- “fractional exclusion statistics”

fractionalization of the h/e vortex:

$$P_0|\ldots 1100110011000110011 0011\ldots\rangle$$

$$P_0|\ldots 11001100101010100110011\ldots\rangle$$

$$P_0|\ldots 1100101010101010101010011\ldots\rangle$$

- lots more, its a great future!