New Zealand INSTITUTE for Advanced Study

Centre for Theoretical Chemistry and Physics

Lecture 2: Semi-topological solitons in multiple dimensions

The Dodd-Walls Centre for Photonics and Quantum Technology

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To be covered: Solitons in quantum gases

- **Lecture 1:** Solitons and topological solitons
  - solitons in water: the KdV equation, iintegrability
  - solitons of the nonlinear Schrodinger equation
  - solitons of the sine Gordon equation - topological solitons
  - Bose Josephson vortices in linearly coupled BECs
- **Lecture 2:** Semitopological solitons in multiple dimension
  - Solitons as quasiparticles: effective mass
  - solitons in the strongly-interacting Fermi gas
  - snaking instability
  - vortex rings
  - solitonic vortices
- **Lecture 3:** Quantum solitons
  - solitons in strongly-correlated 1D quantum gas

Terminology

- **Solitary wave:** localises energy density with constant shape

$$\epsilon(\mathbf{r}, t) = \epsilon(\mathbf{r} - \mathbf{v}t)$$

- **Lump**: localises energy (not always constant shape), e.g. sine-Gordon breather

- **Soliton**, narrow meaning: solitary waves that survive collisions. Wider meaning: any lump or solitary wave

- **Topological soliton**: field solution (mapping) that is distinct from vacuum by homotopy class, e.g. skyrmion.
  Note: No reference to localised character

Skyrmion

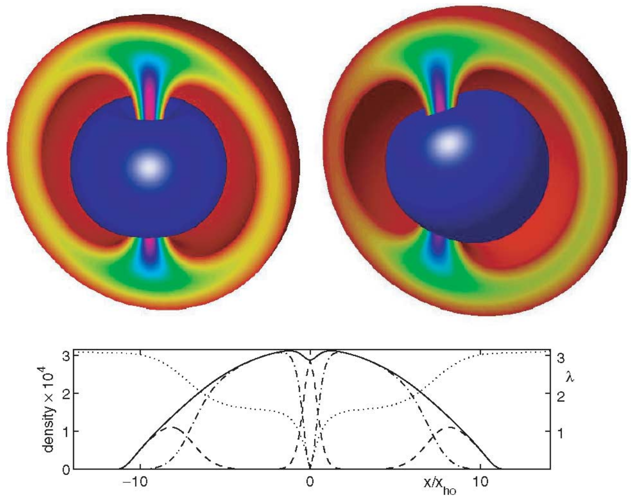
- Originally solution of nonlinear σ-model, topological soliton in the pion field to model low-energy properties of nucleon (explains, e.g. nucleon radius, stability, Tony Skyrme 1961/62).

- Topology: mapping of unit sphere $S^3 \to S^3$
  where $\mathbb{R}^3 \bigcup \{\infty\} \cong S^3$

  Homotopy classes: integer winding numbers
- 1D example:
  sine-Gordon equation $\mathbb{R}^1 \bigcup \{\infty\} \cong S^1 \to S^1$

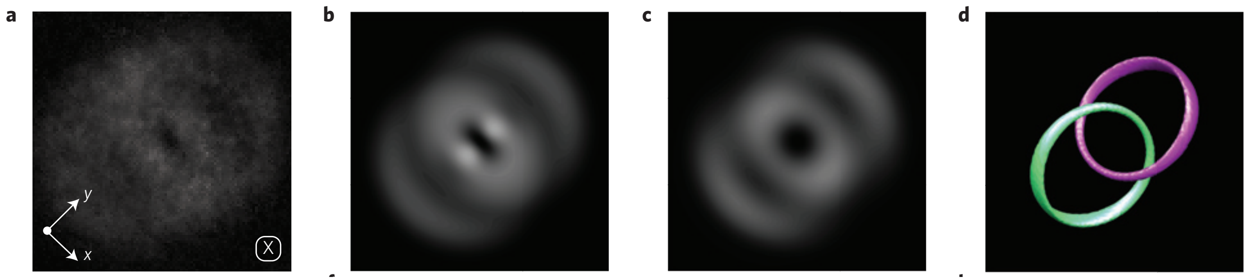
Skyrmions in Bose-Einstein condensate

- BEC with vector order parameter: many proposals (Stoof, Battey, etc. from 2001) but no experimental evidence.
  Problem: stability
  (order parameter may vanish)



Savage, Roustekoski (2003)

- Related experiments by David Hall (Amherst): Dirac monopoles (2015), quantum knots (2016)



Hall et al. (2016)

Superfluid vortex as topological soliton

Vortex in scalar superfluid

$$i\frac{\partial}{\partial t}u(\boldsymbol{r},t) = [-\frac{1}{2}\nabla^2 + |u|^2]u(\boldsymbol{r},t)$$

Velocity field $\boldsymbol{v} \propto -\nabla \arg u$

$$\mathbb{R}^2 \bigcup \{\infty\} \cong S^2 \to S^1$$

Vortices are quantized in the nonlinear Schrödinger equation

$$u(\boldsymbol{r}) = \sqrt{n(r)}e^{i\kappa\varphi}$$ κ = 0, 1, -1, 2, -2, …

Is the vortex a solitary (localised) wave?

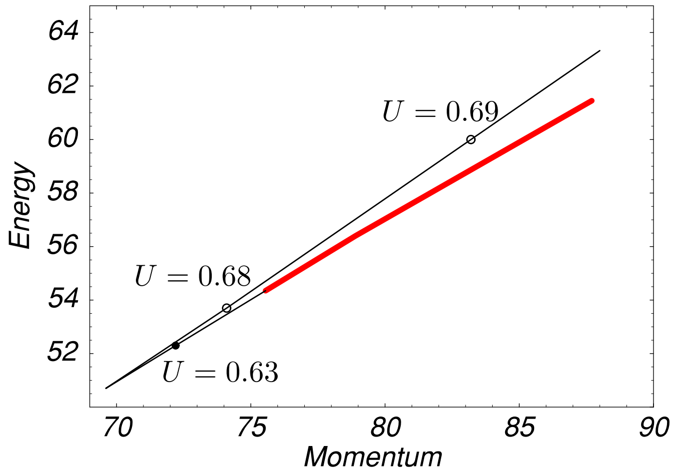
No, it is an extended object.
Even in 2D the energy diverges logarithmically with system size.

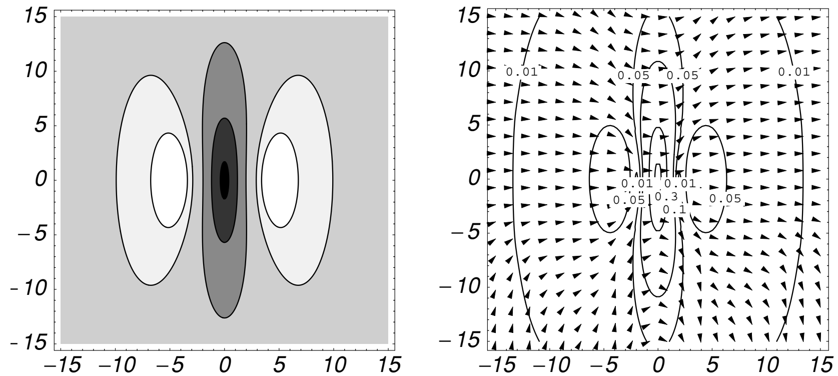
Solitary waves in extended superfluids?

Vortex ring (in 3D) and
Vortex dipole (in 2D):

Are localised *algebraically*

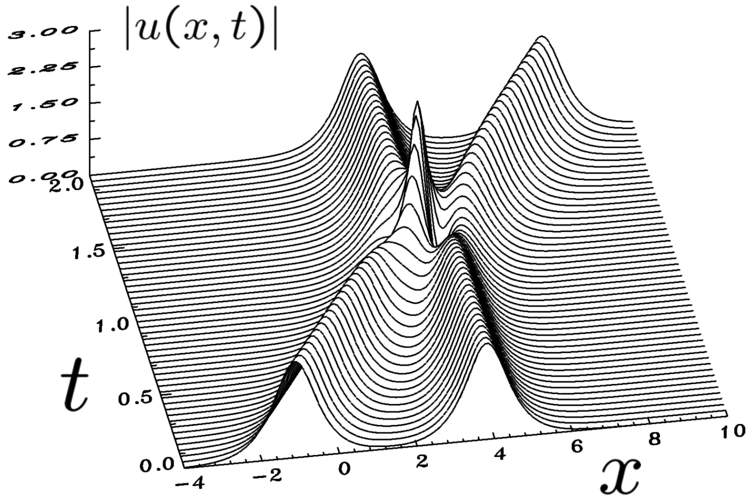
Vortex rings and **rarefaction pulses**
in 3D Gross Pitaevskii equation





Jones and Roberts, JPA (1982), Berloff and Robert JPA (2004)

So, solitons are like particles.
Then, what is the mass?



If solitons are emergent particle-like excitations, their mass is an emergent classical property.

Mass of a ping pong ball under water

Buoyancy force:

$$F_B = mg - m_w g$$
$$\approx -11mg$$

Acceleration:

$$m\ddot{x} = F_B \qquad m^*\ddot{x} = F_B$$

$$\ddot{x} \approx -11g! \qquad \ddot{x} = -g\frac{m_{ph}}{m^*}$$

Physical mass:

$$m_{ph} = m - m_w$$
$$\approx -11m$$

Movie credit: Allan Adams (MIT) et al.

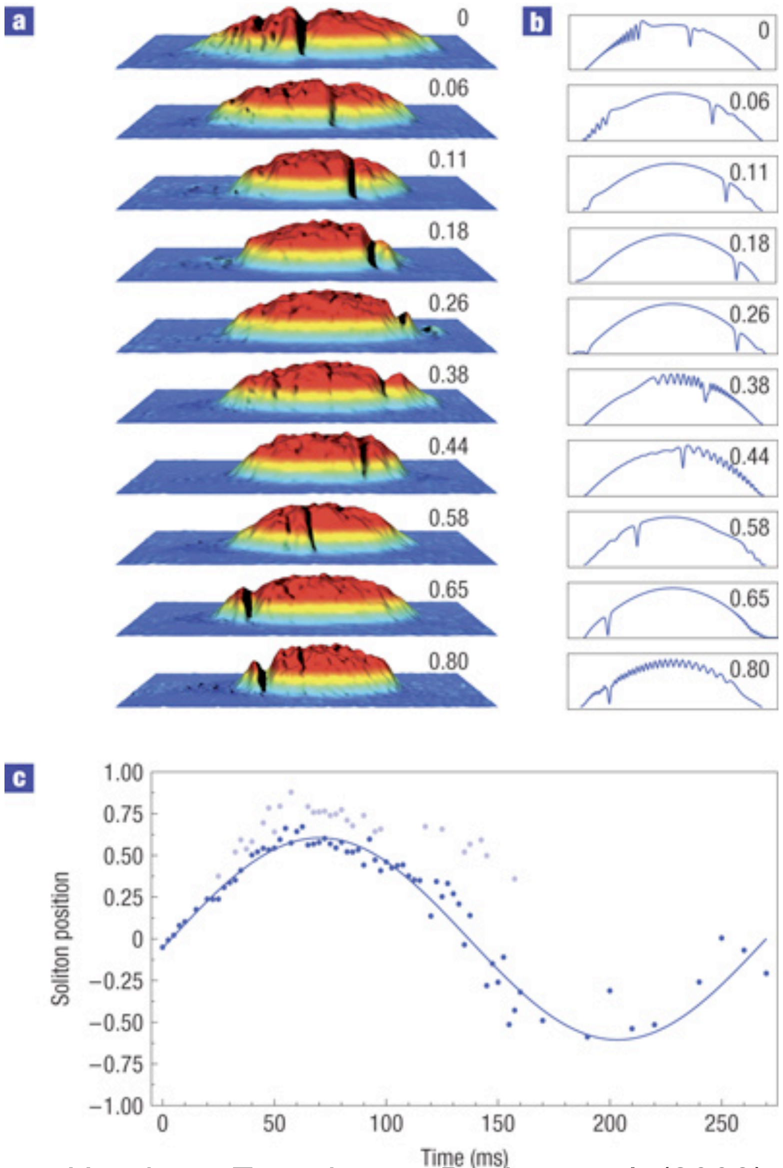
Filmed at 1200fps

Effective (inertial) mass: m^*

Includes mass of water dragged along with the ball

Changes during motion

Dark solitons in a trapped BEC



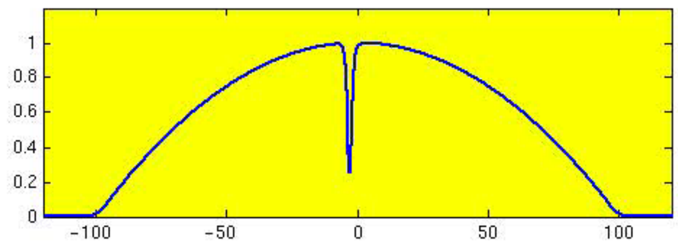
Hamburg Experiment: Becker et al. (2008)

Solitons in trapped BEC oscillate more slowly than COM

$$\left(\frac{T_s}{T_{\text{trap}}}\right)^2 = \frac{m^*}{m_{ph}} = 2$$

Theory:
- Busch, Anglin PRL (2000)
- Konotop, Pitaevskii, PRL (2004)

Experiment:
- Becker et al. Nat. Phys. (2008)
- Weller et al. PRL (2008)



Movie credits: Nick Parker, Univ. Leeds

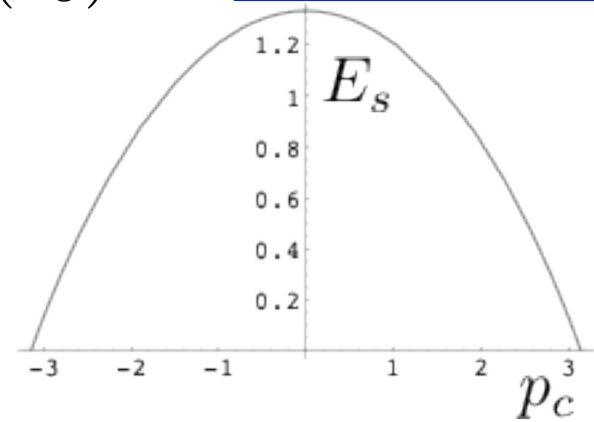
Soliton dispersion

Soliton energy: $E_s(\mu, v_s, g) = \langle \hat{H} - \mu \hat{N} \rangle - E_h$

Canonical momentum: $v_s = \dfrac{dE_s}{dp_c}$

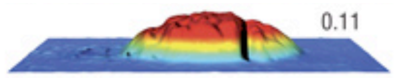
Effective (inertial) mass: $m^* = 2\dfrac{\partial E_s}{\partial (v_s)^2}$

$$\boxed{E_s \approx E_0 + \frac{p_c^2}{2m^*}}$$



Physical (heavy) mass:

$m_{ph} = mN_s$



$N_s = \int (n_s - n_0) d^3 r = -\dfrac{\partial E_s}{\partial \mu}$ (for $v = 0$)

Landau quasiparticle dynamics

Konotop, Pitaevskii, PRL (2004)
Scott, Dalfovo, Pitaevskii, Stringari, PRL(2011)

- soliton moves on a slowly varying background, locally conserving energy

$$\frac{dE_s(v_s,\mu(z))}{dt} = 0 \longrightarrow \text{equation of motion}$$

- For *harmonic trapping potential* obtain small amplitude oscillations with

$$\left(\frac{T_s}{T_{\text{trap}}}\right)^2 = \frac{m^*}{m_{ph}}$$

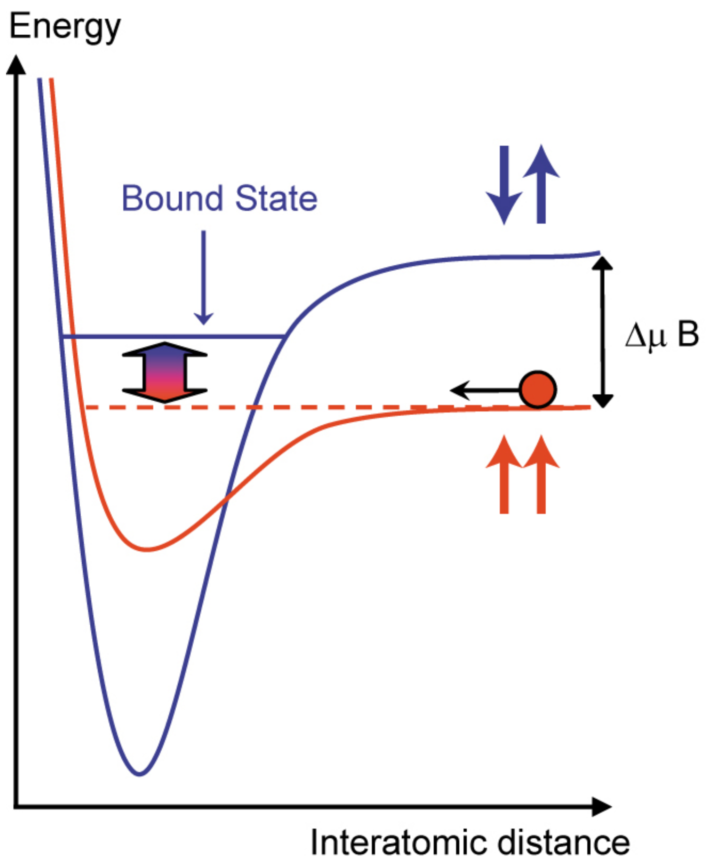
  - BEC solitons: also locally conserve particle number

$$\frac{m^*}{m_{ph}} = 2 \qquad\qquad N_s = f(E_s(v_s,\mu))$$

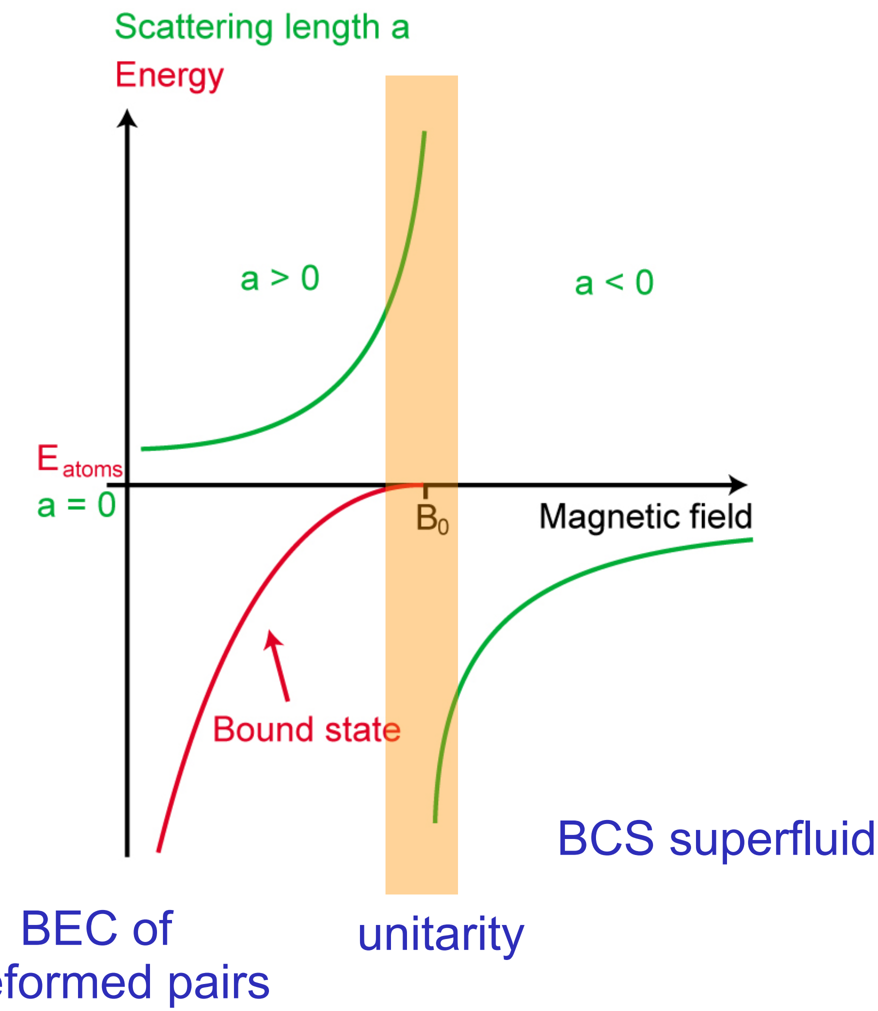
What about dark solitons in a fermionic superfluid?

We only need to compute the dispersion relation
to obtain the mass ratio and predict oscillation frequencies …

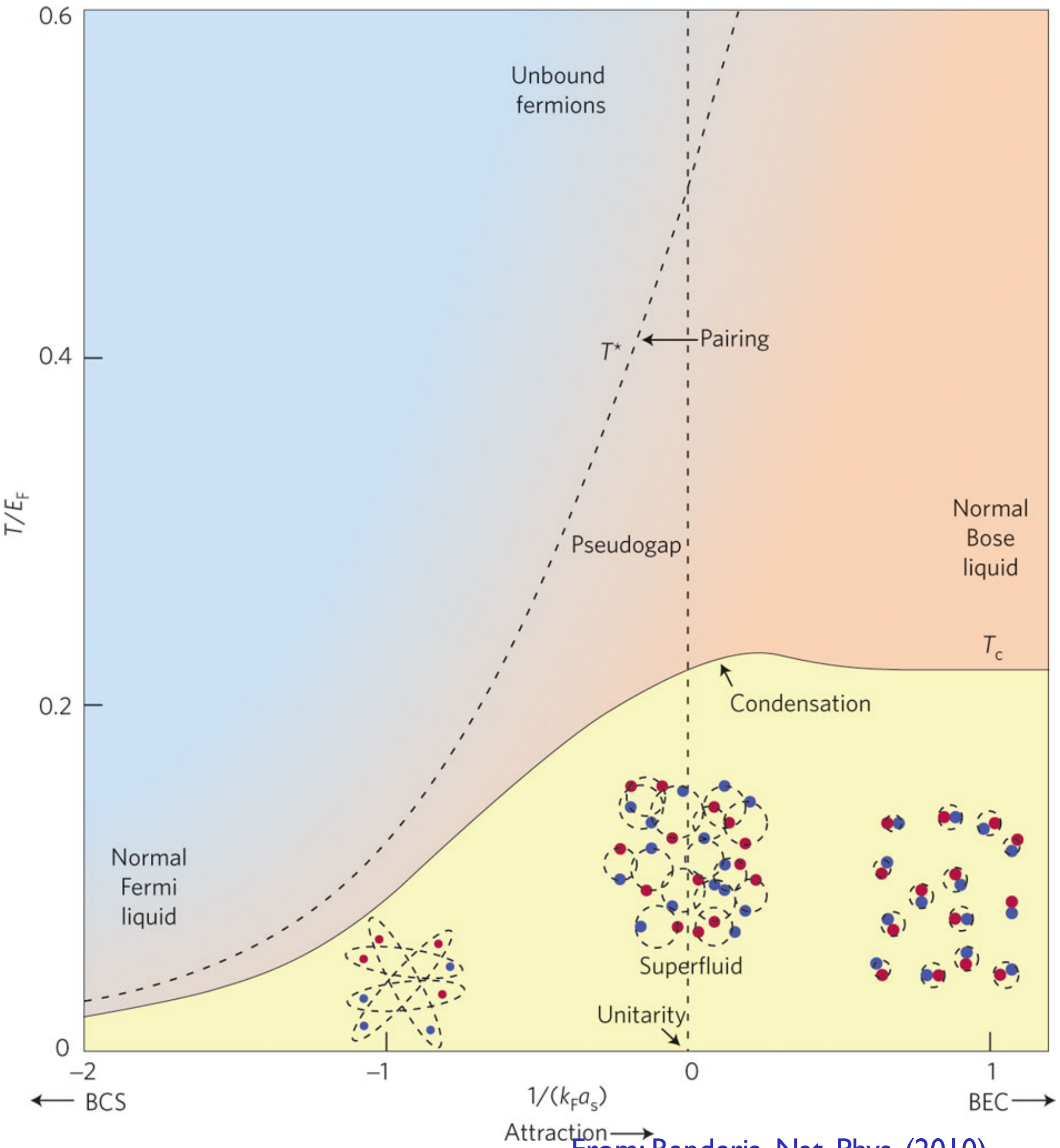
Feshbach resonance for spin-1/2 fermions



credit: MIT group



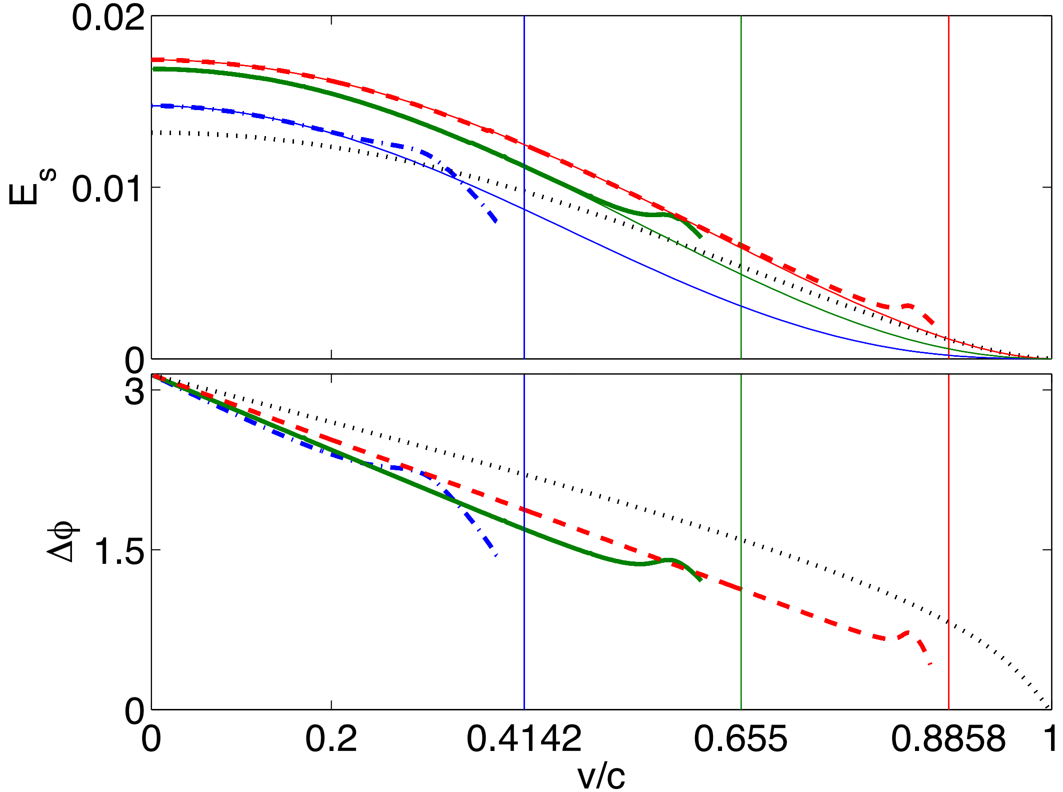
BEC to BCS crossover Fermi gas



From: Randeria, Nat. Phys. (2010)

Can solitons probe strongly-interacting physics beyond hydrodynamics?

Dispersion relations: computed from Bogoliubov-de Gennes equation



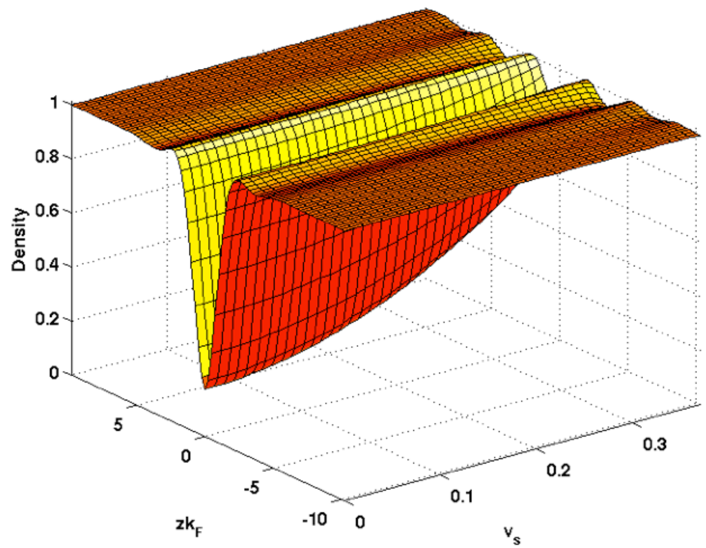
lines: fit of $(1 - v^2/c^2)^\alpha$

unitarity : $1/k_F a = 0$

BCS(green) : $1/k_F a = -0.2$

BEC(dotted) : $1/k_F a = 1$

BCS(blue) : $1/k_F a = -0.5$



Termination points reveal fermionic physics.

Liao, Brand PRA 83, 041604(R) (2011)

Scott, Dalfovo, Pitaevskii, Stringari, Fialko, Liao, Brand NJP 14, 023044 (2012)

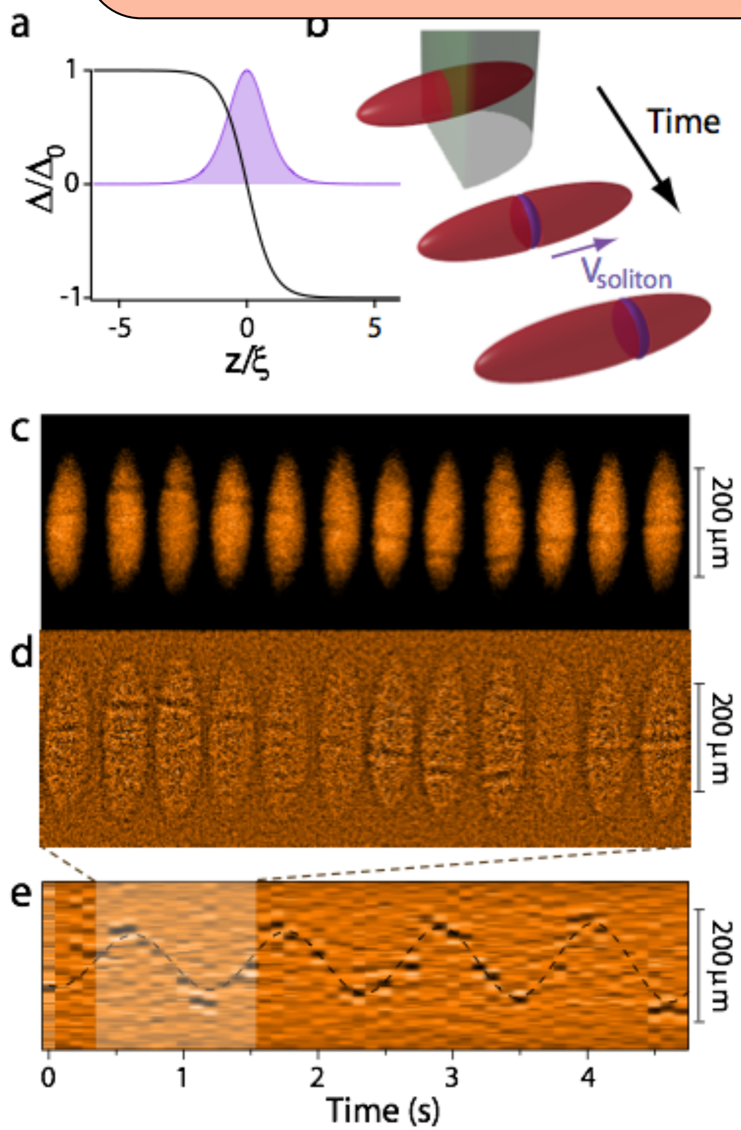
- Nature 2013:

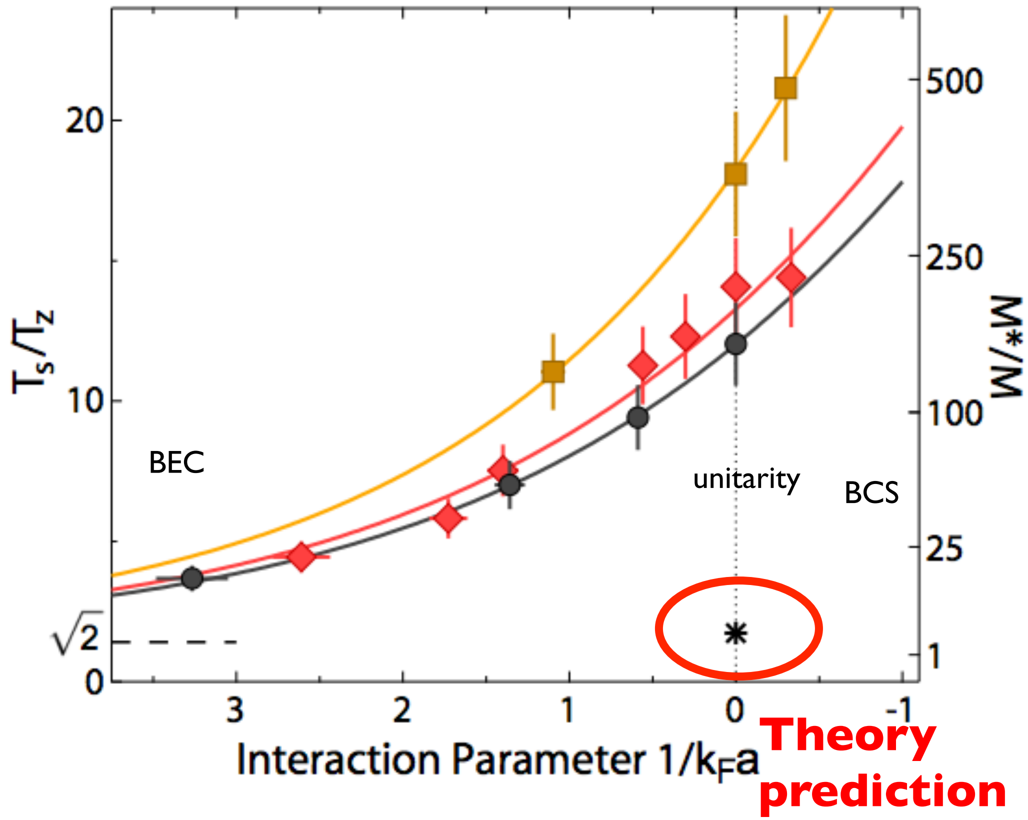

ARTICLE

doi:10.1038/nature12338

Heavy solitons in a fermionic superfluid

Tarik Yefsah[1], Ariel T. Sommer[1], Mark J. H. Ku[1], Lawrence W. Cheuk[1], Wenjie Ji[1], Waseem S. Bakr[1] & Martin W. Zwierlein[1]





Experiment: Yefsah et al., Nature (2013)
Theory: Liao, Brand, PRA (2011),
Scott, Dalfovo, Pitaevskii, Stringari, PRL (2011)

Resolution of the riddle: solitonic vortex

- Nature 2013:

ARTICLE

doi:10.1038/nature12338

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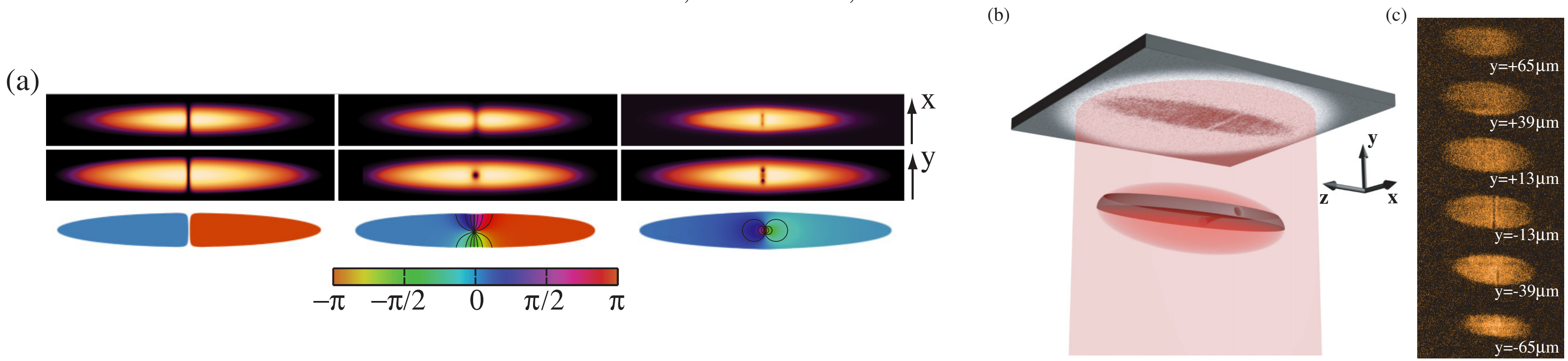
Slowly oscillating solitons in trapped Fermi superfluid turn out to be ***solitonic vortices.***

Selected for a Viewpoint in *Physics*

PRL **113,** 065301 (2014) PHYSICAL REVIEW LETTERS week ending 8 AUGUST 2014

Motion of a Solitonic Vortex in the BEC-BCS Crossover

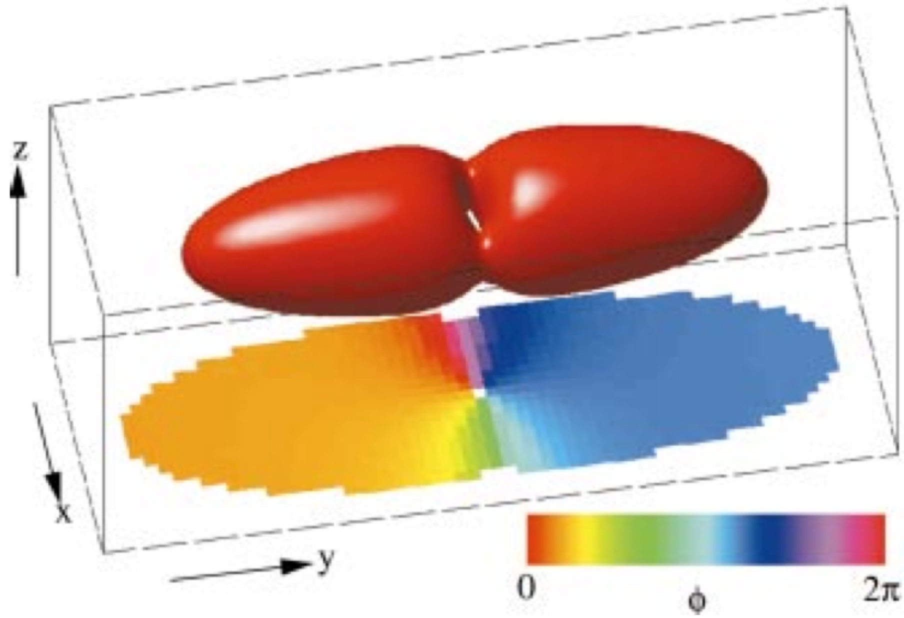
Mark J. H. Ku, Wenjie Ji, Biswaroop Mukherjee, Elmer Guardado-Sanchez, Lawrence W. Cheuk, Tarik Yefsah, and Martin W. Zwierlein



What is a solitonic vortex?

- … a solitary wave that is localised (exponentially) in the long dimension of a fluid that is confined in the other two dimensions.
- … a single vortex filament.

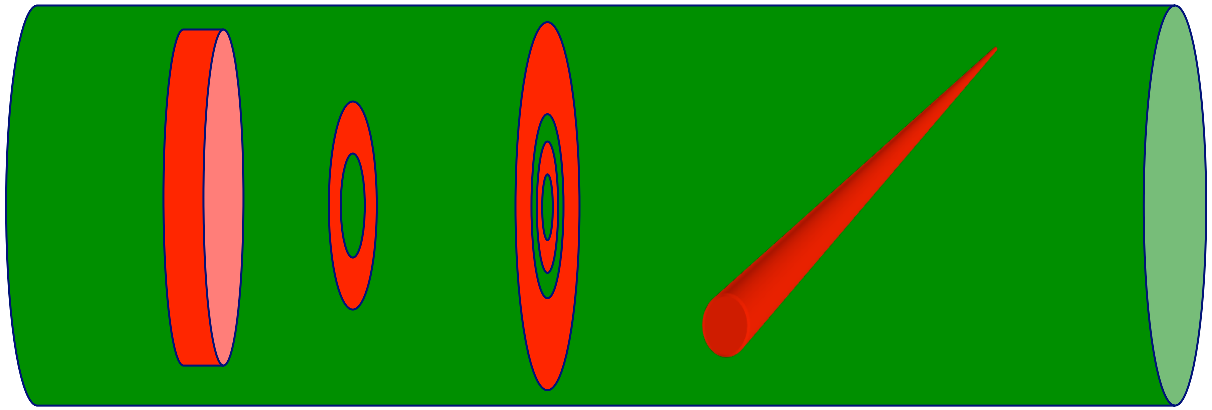
J.B., W.P. Reinhardt, JPB 37, L113 (2001)
J.B., W.P. Reinhardt, PRA 65, 043612 (2002)



Solitary waves in 3D waveguides

axially symmetric

not axially symmetric



planar soliton

solitonic vortex

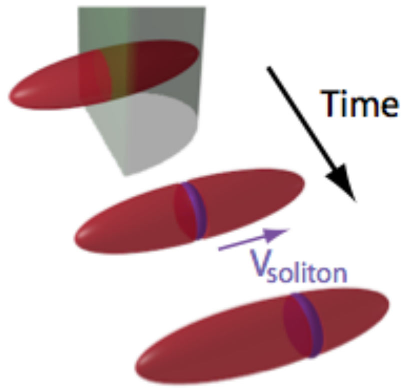
vortex ring

double ring

more ...

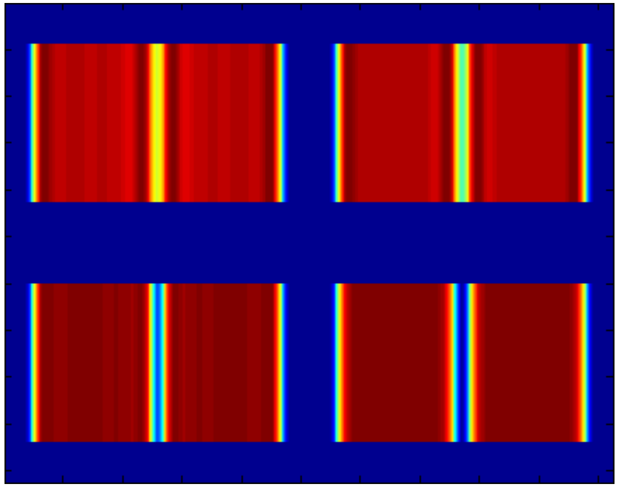
How do solitonic vortices form?

Phase imprinting generates dark soliton



But: dark soliton is unstable with respect to the snaking instability

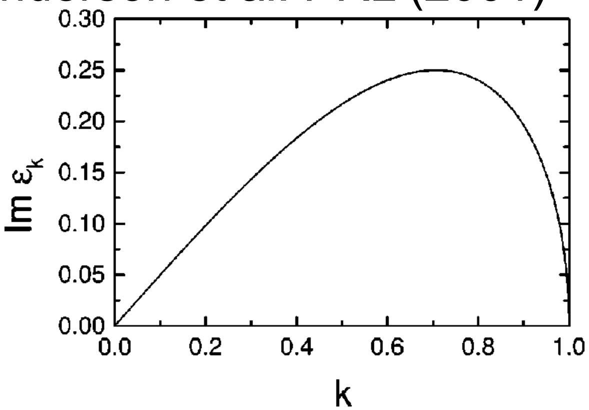
Snaking instability for homogeneous Fermi gas

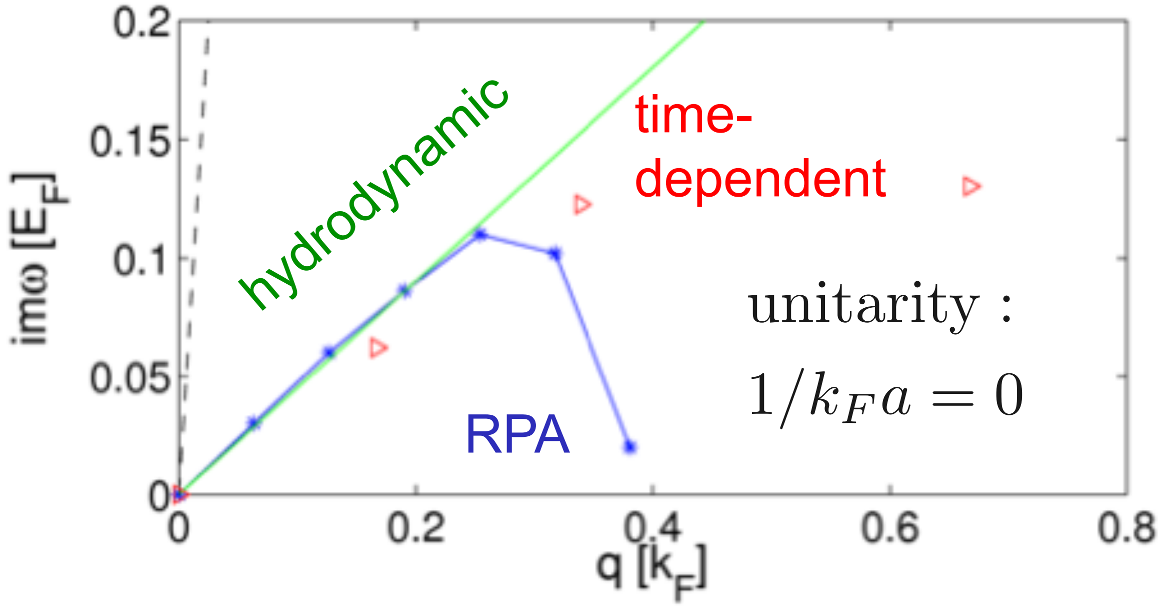


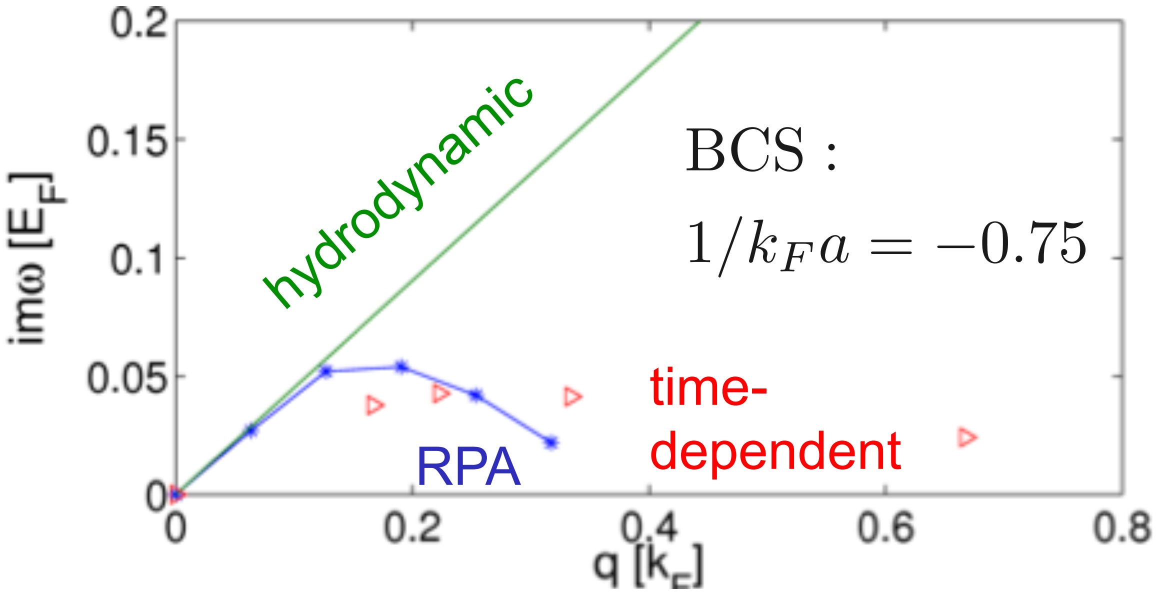
BEC (theory):
Kuznetsov and Turitsyn JETP (1988) Muryshev et al. PRA (1999)

- BEC experiment

Anderson et al. PRL (2001)







Cetoli, Brand, Scott, Dalfovo, Pitaevskii, PRA (2013)