Lecture II: Hadronic Gamma-Rays

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Motivating question: why is the Galactic plane so bright in γ-rays?

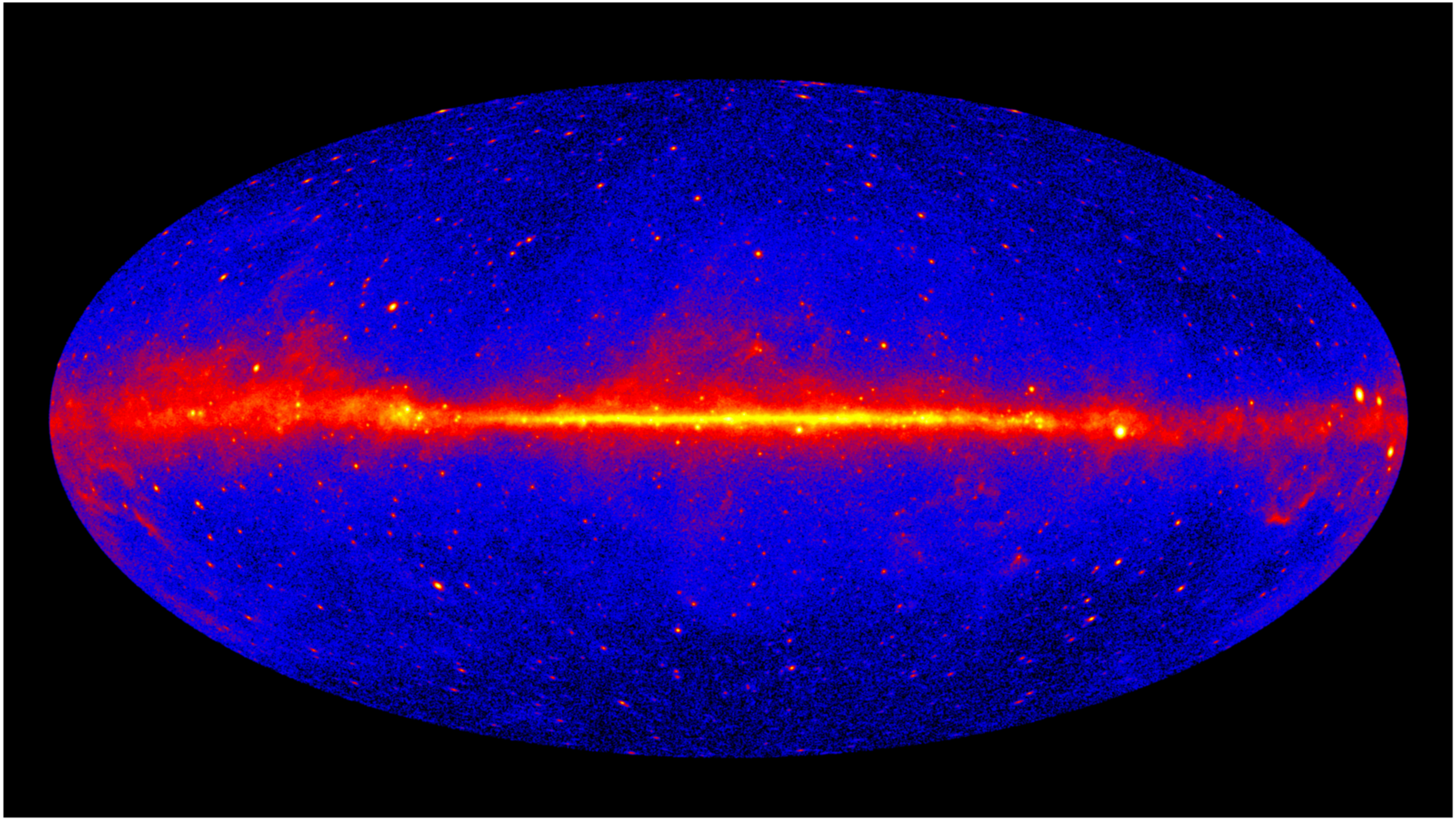


Figure 1: *Fermi*-LAT all sky image in Galactic co-ordinates. Credit: NASA/DoE.

Motivating question: why is the Galactic plane so bright in γ-rays?

- The previous figure displays an image created with data collected with NASA's orbiting Femi-LAT γ-ray telescope (operating in the ~ 100 MeV – 1 TeV range).
- The image is in Galactic coordinates where the Galactic centre is in the middle and the Galactic plane runs horizontally (Galactic east is to the left of the centre and Galactic west to the right).
- The false colours trace the intensity of the integrated γ-ray flux coming from each direction of the sky. Evidently, the **Galactic plane glows brightly in γ-rays**. In this lecture we will learn about the microphysical process responsible for the lion's share of this γ-ray emission.

Motivating question: why is the Galactic plane so bright in γ-rays?

- **Spoiler**: for the Galactic plane, most of the γ-ray production is diffuse in nature – i.e., not associated with individual, point-like γ-ray sources – and happens as a result of the collision of cosmic ray ions with diffuse ISM gas.
- Here ions means protons and heavier, ionised nuclei (He, C, N, O, etc). For this reason, this mechanism is called '**hadronic**' γ-ray production as it is associated with collisions of hadrons – in this case, protons and neutrons (within the nuclei of the 'beam' cosmic ray ions) with protons and neutrons within the 'target' ISM gas. These collisions are mediated by the strong nuclear force (whose dynamics are governed by Quantum Chromodynamics).
- γ-ray production processes associated with cosmic ray electrons (and / or the positrons) – inverse Compton (IC) emission and bremstrahlung emission – make a sub-dominant but still important contribution to Galactic plane emission. Generically, because they involve cosmic ray electron (or positrons) beam particles, IC and bremsstrahlung are referred to as '**leptonic**' γ-ray production processes; these processes do not involve the strong force, which is not experienced by electrons.

Processes of relevance to hadronic interactions

- Let us consider the sorts of processes relevant to hadronic interactions in astrophysics.
- Such interactions are identical to the those that are generated at the Large Hadron Collider (LHC) near Geneva.
- At the LHC, counter-propagating beams of protons are smashed into each other at high Centre-of-Momentum energies $\sqrt{s} \sim 14$ TeV.

Processes of relevance to hadronic interactions

Hadronic interactions

- In deciding whether certain reactions are allowed we can use the following principles:
  - momentum/energy must be conserved;
  - baryon number must be conserved;
  - lepton number must be conserved (in addition, lepton flavor is usually conserved);
  - electric charge must be conserved.
- [For the moment, we simply assume that we have sufficient energy in the Centre-of-Momentum frame to create the secondary particles in the reactions below].

Hadronic interactions

- In passing, note that most (baryonic) matter in the Cosmos is 'hydrogen', i.e. in the form of a proton which may be in molecular (H2), atomic (HI), or ionized (HII) form (i.e., single protons).
- Here we are typically interested in processes initiated by relativistic protons (or heavier ions); this implies that the relevant particles have kinetic energies comparable to or larger than their rest mass energies.
- You should keep in your head that protons have rest mass energies of ~ GeV = 10^9 eV, so the kinetic energies of the protons (or heavier ions) under consideration here satisfy $T_p \gtrsim$ GeV (in contrast, the rest mass of electrons is a factor of 1000 smaller at ~MeV).
- Given, the $\gtrsim$ GeV energy scale of the beam protons we consider is $\gg$ the ~ eV scale of molecular/atomic bindings and also given that, at this same large energy scale, the beam protons are hardly deflected by the electromagnetic charge of target protons (were they ionised), it is **completely irrelevant** here whether the target matter is molecular, atomic, or ionised hydrogen gas.

Hadronic interactions

- Here are some possible inelastic 'pp' processes that occur in the collision of a high-energy, relativistic 'beam' cosmic ray proton with ambient gas:

$$
\begin{aligned}
p + p &\rightarrow p + p + \pi^0 \\
p + p &\rightarrow p + n + \pi^+ \\
p + p &\rightarrow p + p + \pi^+ + \pi^- \\
p + p &\rightarrow p + p + \pi^+ + \pi^- + \pi^0
\end{aligned}
$$

- On the right side of each reaction you will see one or more pions in addition to two baryons (either p + p or p + n).

Hadronic interactions

- Pions are the lightest mesons (and, in fact, the lightest baryons).
- Pions are composed of various (quantum mechanical) combinations of u and d quark pairs such that they posses no overall baryon number ($B(\pi) = 0$).
- Thus all these interactions conserve baryon number: there are two hadrons on both the LHS and the RHS of each of the interactions in the list above ($B(p) = B(n) = 1$).
- Also, each interaction conserves electric charge (check).
- Overall, as the Centre-of-Momentum energy increases, pp interactions tend to produce more and more mesons; these will be mostly pions, be will also include subdominant components of Kaons K (that contain strange quarks) and even heavier 'charmed' mesons.
- Given sufficient energy in the Centre-of-Momentum, we can also produce particle-anti-particle pairs: e.g., proton and anti-proton ($p\bar{p}$).
- Much more rarely, such interactions can also produce (non-virtual) gauge bosons and even the Higgs particle (but all such particles – with the exception of the photon – are unstable and decay very quickly).

Hadronic interactions

- The three types of pion and their valence quark content are:

$$\begin{array}{ll} \pi^0 & (u\bar{u} \text{ or } \mathrm{d}\bar{\mathrm{d}}, \sim 135 \text{ MeV}) \\ \pi^+ & (u\bar{d}, \sim 140 \text{ MeV}) \\ \pi^- & (d\bar{u}, \sim 140 \text{ MeV}) \end{array}$$

- Note that the π^+ and π^- have the same mass (WHY?) and, once in the regime well above the threshold for inelastic pp collisions (see below), the three species of pion are produced in approximately equal numbers by pp collisions.
- Because they possess no overall baryon number, all mesons, including the pion, are unstable.
- The π^0 decays extremely quickly (8.4×10^{-17} s) into two photons in a process mediated by the electromagnetic force.

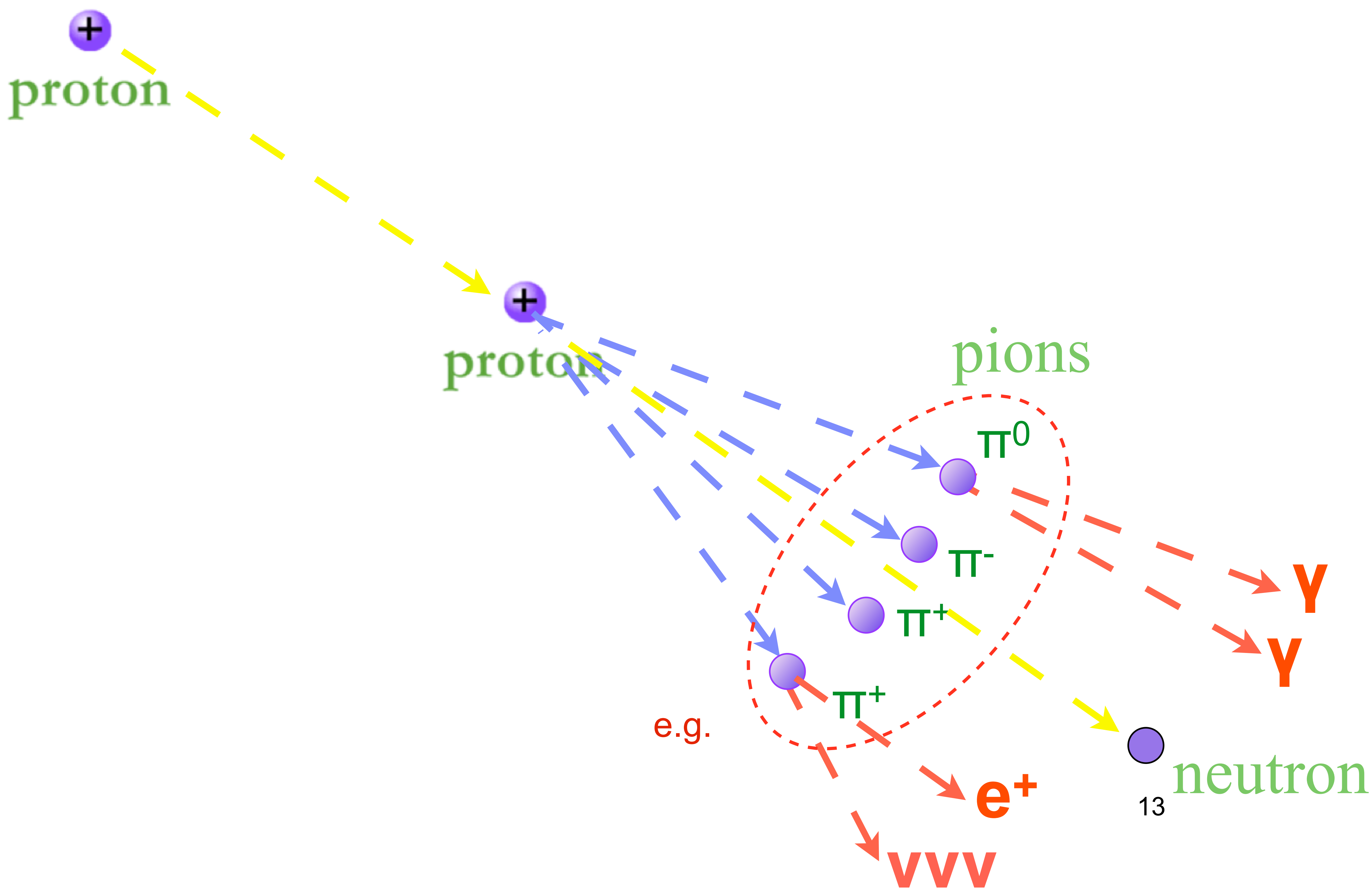
Hadronic interactions

- The charged pions decay more slowly (2.6×10^{-8} s) via the weak force into (mostly), first, muons and accompanying mu neutrinos; the muons then subsequently decay to electrons/positrons and further neutrinos (so as to conserve overall lepton flavor).

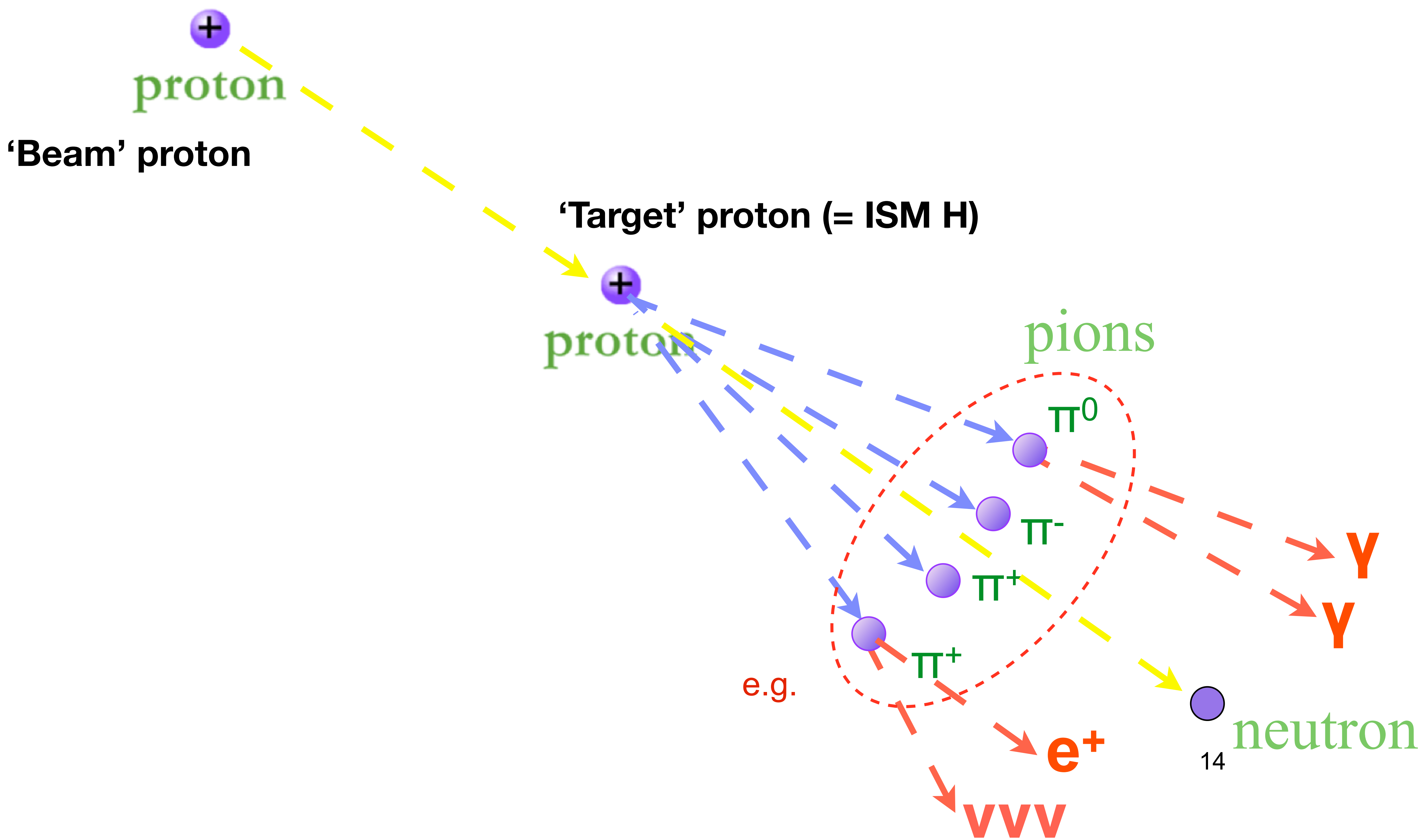
$$
\begin{array}{lllll}
\pi^0 & \rightarrow & \gamma + \gamma & & \\
\pi^+ & \rightarrow & \mu^+ + \nu_\mu & \text{then} & \mu^+ \rightarrow \mathrm{e}^+ + \nu_\mathrm{e} + \bar{\nu}_\mu \\
\pi^- & \rightarrow & \mu^- + \bar{\nu}_\mu & \text{then} & \mu^- \rightarrow \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu
\end{array}
$$

- Note that the first channel above produces photons; in the rest frame of the parent π^0 each photon carries away equal and opposite 3-momentum and therefore has equal energy $E_\gamma = m_{\pi^0}/2 \simeq 67$ MeV. This energy scale is well into the γ-ray regime (defined roughly by $E_\gamma \gtrsim 1$ MeV).
- Thus pp collisions lead to the (indirect) production of γ-rays (!) and it is this process that dominates the production of the diffuse γ-ray emission from the Galactic plane seen above.

pp → pion decay → secondaries



pp → pion decay → secondaries



Hadronic interactions

$$
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\end{array}
$$

- In this lecture, we will consider the consequences of neutral pion production and subsequent decay.
- In particular, pp collisions lead to the (indirect) production of γ-rays (!) and it is this process that dominates the production of the diffuse γ-ray emission from the Galactic plane seen above.
- In the next lecture, we will consider the consequences of charged pion production and decay.

Hadronic gamma-ray production

Hadronic gamma-ray production

- From elementary kinematics, we can work out that the threshold proton kinetic energy T_p to produce a neutral pion via $p + p \rightarrow p + p + \pi^0$ is 280 MeV
- Thus 'beam' CR protons must be at least trans-relativistic to generate hadronic gamma-rays off gas in the ISM
- This is reflected in the energy dependence of the pp ***cross-section***

Hadronic cross-section σ_{pp}

- A cross-section, which has units of area, is basically an expression of the interaction probability for a given beam particle to interact within a given population of target particles.
- Specifically, say our beam particles are protons of velocity v which are impinging on a gas of pure hydrogen of volumetric number density n_H, then the rate R_{pp} at which each beam particle experiences collisions with the target gas is given by $R_{pp} = v\sigma_{pp}n_H$
- This equation has the expected behaviour: the RHS has units of t^{-1} and R_{pp} increases linearly with the velocity of the beam particles, the size of the pp cross-section, and the target particle number density as one would intuit. Note that $v \rightarrow c$ for relativistic particles.

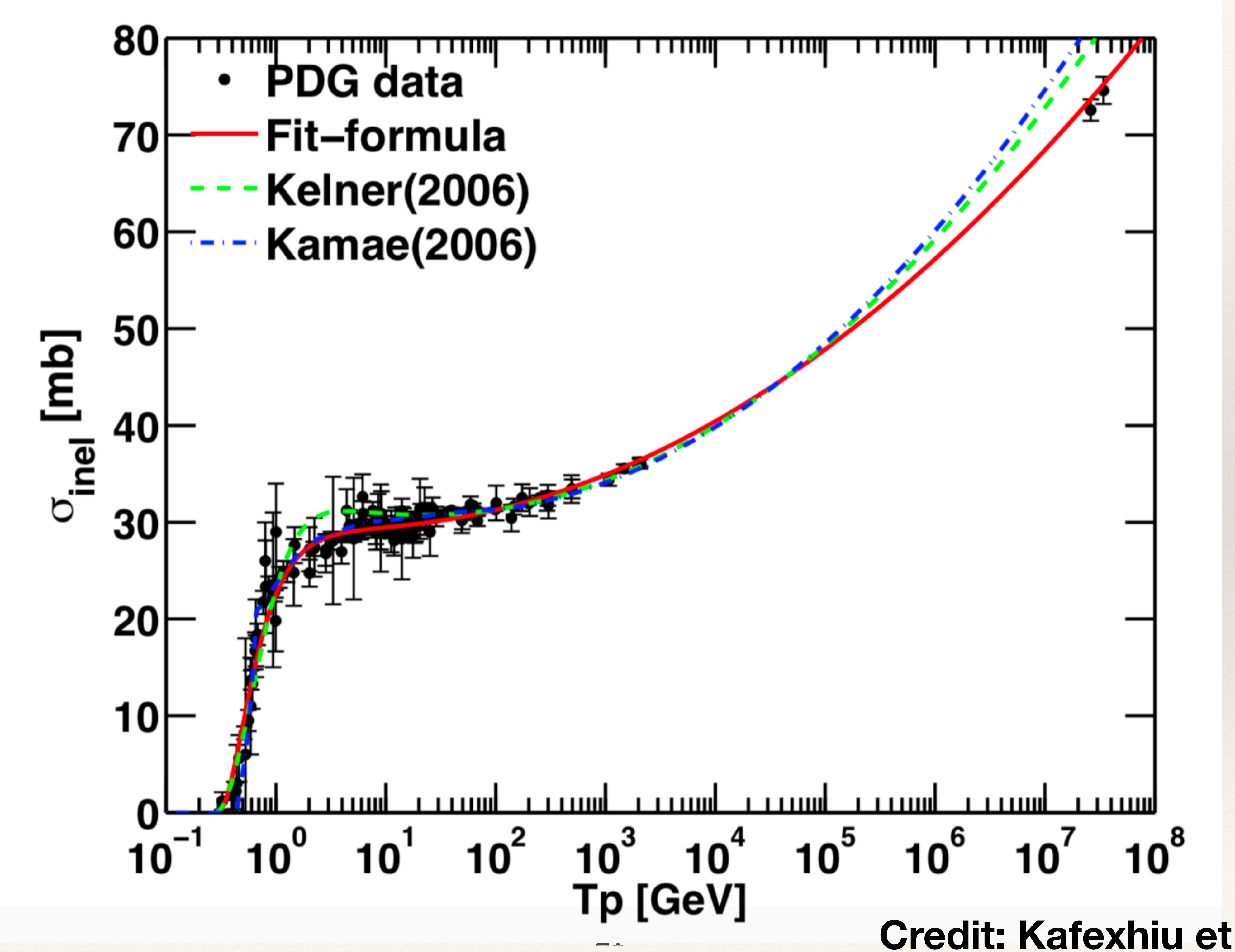
Hadronic cross-section σ_{pp}

- Here σ_{pp} represents the total cross-section, i.e., the sum of the elastic (no new particles created in the interaction, though energy/momentum may be redistributed between target and beam) and inelastic cross-sections (where at least one new particle is created).
- You will see reference made to **inclusive** cross-sections, e.g., the inclusive cross-section for $pp \to \pi^0$, denoted $\sigma_{pp \to \pi^0 X}$; this represents a quantification for the likelihood that the beam proton collides with a target proton to create a π^0 and anything else, as denoted by X.

Hadronic cross-section σ_{pp}

- For convenience, to measure cross-sections particle physicists use an area unit of "barns", 1 b = 10^{-24} cm^2 or, for hadronic cross-sections, millibarns, mb = 10^{-27} cm^2 are often used.
- Just above the pion production threshold of $T_{p,thresh} \simeq 280$ MeV mentioned above, the inelastic pp cross-section grows quickly, but this quickly saturates such that above beam proton kinetic energies ~ few GeV, the inelastic cross-section grows quasi logarithmically (i.e., slowly) to a few 10s of mb.

Hadronic cross-section σ_{pp}



Credit: Kafexhiu et al. (2014)

Hadronic cross-section σ_{pp}

- We can also define an inelasticity κ; this quantifies the amount of energy the beam particle typically loses in each collision. Thus $\kappa_{pp} \simeq 0.5$ roughly describes the fraction of energy that a beam proton loses in each hadronic collision.
- The combination of cross-section and inelasticity allows us to calculate a characteristic loss time due to collisions (the timescale over which the beam particle's energy drops to 1/e of its initial energy). For pp collisions this timescale is:

$$t_{pp} \equiv \frac{1}{v\sigma_{pp}\kappa_{pp}n_H}.$$

Spectrum of hadronic gamma-rays

Spectrum of hadronic gamma-rays

- Let us focus on the simplest and most important channel for such hadronic γ-ray production in the interstellar medium: $p + p \rightarrow \pi^0 + X \rightarrow 2\gamma + X$ (remember that each π^0 decays into two γ-ray photons)
- To deal with this two-step process, we break our calculation into two parts addressing
  - i) the spectrum of γ-rays produced by the decay of a given distribution of π^0 and
  - ii) the production of π^0 by pp collisions.
- Finally, we knit these together to work out the spectrum of γ-rays emerging from the collisions experienced by a given population of cosmic ray protons.
- Note that we will use a unit system $c = 1$ so that masses, energies and momenta all have the same units.

Spectrum of hadronic gamma-rays

- First, let $dQ_\gamma(E_\gamma)/dE_\gamma$ denote the (differential) volumetric emissivity of γ-rays due to π^0 decay. This is a quantity measuring the number of γ-rays in some energy bin

$E_\gamma \to E_\gamma + \delta E_\gamma$

produced per unit time and per unit volume by such decays, i.e., a quantity with units of (e.g.,) $cm^{-3}\ s^{-1}\ eV^{-1}$.

(Below we denote quantities measured in the centre-of-momentum frame with an asterisk.)

- The differential emissivity of γ-rays from π^0 decay may be written:

$$\frac{dQ_\gamma(E_\gamma)}{dE_\gamma} = \int_{E_\pi^{min}(E_\gamma)} dE_\pi \frac{dP(E_\pi \to E_\gamma; E_\pi, E_\gamma)}{dE_\gamma} \frac{dQ_\pi(E_\pi)}{dE_\pi}$$

Spectrum of hadronic gamma-rays

$$\frac{dQ_\gamma(E_\gamma)}{dE_\gamma} = \int_{E_\pi^{min}(E_\gamma)} dE_\pi \frac{dP(E_\pi \to E_\gamma; E_\pi, E_\gamma)}{dE_\gamma} \frac{dQ_\pi(E_\pi)}{dE_\pi}$$

- $dP(E_\pi \to E_\gamma; E_\pi, E_\gamma)/dE\gamma$ is the differential probability (probability density) that a π of E_π decays to produce a γ of E_γ and $E_\pi^{min}(E_\gamma)$ is the minimum E_π required to produce a γ of E_γ
- hereafter we drop the '0' superscript on the pion label for simplicity; π should be understood to mean π^0 unless indicated otherwise

Spectrum of hadronic gamma-rays

- To determine $E_\pi^{min}(E_\gamma)$ the minimum pion energy required to produce a γ-ray detected in the lab frame with E_γ, first use conservation of 4-momentum (squared), P , where $P^2 \equiv s$: $s_{before} = s_{after}$:

$$s_{before} = s_\pi = P_\pi^2 = m_\pi^2$$

$$s_{after} = (P_{\gamma_1} + P_{\gamma_2})^2 = 2P_{\gamma_1}.P_{\gamma_2} = 2E_{\gamma_1}E_{\gamma_2}(1 - \cos\Theta)$$

where we have used the fact $P_\gamma^2 \equiv 0$ given photons are massless and Θ is the angle between the photons' 3-momentum vectors.

- Now use frame invariance of s: go to the π rest frame which is identical to the CoM frame: conservation of 3-momentum implies that, in this frame, the two γ-rays γ_1 and γ_2 have equal and opposite 3-momenta, therefore the angle between their momentum vectors in the CoM is therefore $\Theta_\star = 180^\circ$, and $E_{\gamma 1}^\star = E_{\gamma 2}^\star \equiv E_\gamma^\star$. Then we have:

$$m_\pi^2 = 4(E_\gamma^\star)^2 \Rightarrow E_\gamma^\star = \frac{m_\pi}{2}$$

Spectrum of hadronic gamma-rays

- The threshold pion energy to produce a photon at E_γ, $E_\pi^{min}(E_\gamma)$, will correspond to the maximum blue shift, i.e., a π propagating directly at the observer one of whose decay photons is emitted directly at the observer.

- To work this out, we need to boost from the π rest frame back to the lab (observer) frame in which the pion is measured to have Lorentz β_π and γ_π. In general, for the boost we have:

$$\begin{aligned} E_\gamma^{(lab)} &= \gamma_\pi E_\gamma^\star + \beta_\pi \gamma_\pi p_\gamma^\star \cos\theta^\star \\ &= \gamma_\pi E_\gamma^\star (1 + \beta_\pi \cos\theta^\star) \end{aligned}$$

where $\theta^\star$ is angle in the CoM between the propagation direction of one of the γ-rays and the observer.

Spectrum of hadronic gamma-rays

- So, for the particular case $\theta^\star = 0$ (for maximum blueshift at threshold) we have $E_\gamma^{lab}(\theta^\star = 0) \quad = \gamma_\pi E_\gamma^\star (1 + \beta_\pi)$
- Now substitute in the Lorentz γ_π and β_π factors for the pion (measured in the lab frame) to transform from the CoM (= π^0 rest frame) to the observer/lab frame:

$$
\begin{aligned}
\beta_\pi &= \frac{p_\pi}{E_\pi} = \frac{\sqrt{E_\pi^2 - m_\pi^2}}{E_\pi} \\
\gamma_\pi &= \frac{E_\pi}{m_\pi} \\
E_\gamma^\star &= \frac{m_\pi}{2}
\end{aligned}
$$

Spectrum of hadronic gamma-rays

- We find: $$E_\gamma^{(lab)}(\theta^\star = 0) = \frac{1}{2}(E_\pi + \sqrt{E_\pi^2 - m_\pi^2})$$

- …which we can invert to get: $$E_\pi^{min}(E_\gamma) = E_\gamma + \frac{m_\pi^2}{4E_\gamma}$$

Spectrum of hadronic gamma-rays

- In the π^0 rest frame, decay γ's will always have energy $m_\pi/2$. However, even keeping the initial energy of the π^0 fixed in the observer frame, the π^0-decay γ-rays will be detected by the observer to span a range of energies because of the different directions in which the π decay axis can lie with respect to the observer.
- From the previous equation

$$\begin{aligned} E_\gamma^{(lab)} &= \gamma_\pi E_\gamma^\star + \beta_\pi \gamma_\pi p_\gamma^\star \cos\theta^\star \\ &= \gamma_\pi E_\gamma^\star (1 + \beta_\pi \cos\theta^\star) \end{aligned}$$

we find at fixed E_π: $$dE_\gamma^{(lab)} = \gamma_\pi E_\gamma^\star \beta_\pi d(\cos\theta^\star)$$

Spectrum of hadronic gamma-rays

- In the absence of polarization effects, we must have that, in the pion rest frame, there is no privileged direction, i.e., decay into any direction is equally likely. So the number of gamma rays radiated into any small solid angle $d\Omega^\star$ around any direction in the sky (specified in the CoM frame) is a constant. Indeed, given there are 2 γ's radiated into the 4π solid angle of the entire sky from each decay, we have: $$\frac{dn_\gamma}{d\Omega^\star} = const = \frac{2}{4\pi}\,.$$

Spectrum of hadronic gamma-rays

- But we also have from previous:

$$\begin{aligned} d\Omega^\star &= const = 2\pi d(\cos\theta^\star) \propto dE_\gamma^{(lab)} \\ \Rightarrow \frac{dn_\gamma}{dE_\gamma^{(lab)}} &= const = \frac{2}{\beta_\pi\gamma_\pi m_\pi} = \frac{2}{p_\pi} = \frac{2}{\sqrt{E_\pi^2 - m_\pi^2}} \end{aligned}$$

- Furthermore, we know the bounds on $E_\gamma^{(lab)}$ as given by $\theta^\star : 0 \rightarrow 180^\circ$:

$$\gamma_\pi(1-\beta_\pi)E_\gamma^\star \leq E_\gamma^{(lab)} \leq \gamma_\pi(1+\beta_\pi)E_\gamma^\star$$

(where the minimum occurs for the case that the γ-ray emission in the π^0 rest frame is directly away from the observer and the maximum is for emission towards the observer as explored above)

Spectrum of hadronic gamma-rays

- Or, using $\gamma \equiv 1/\sqrt{1-\beta^2}$:

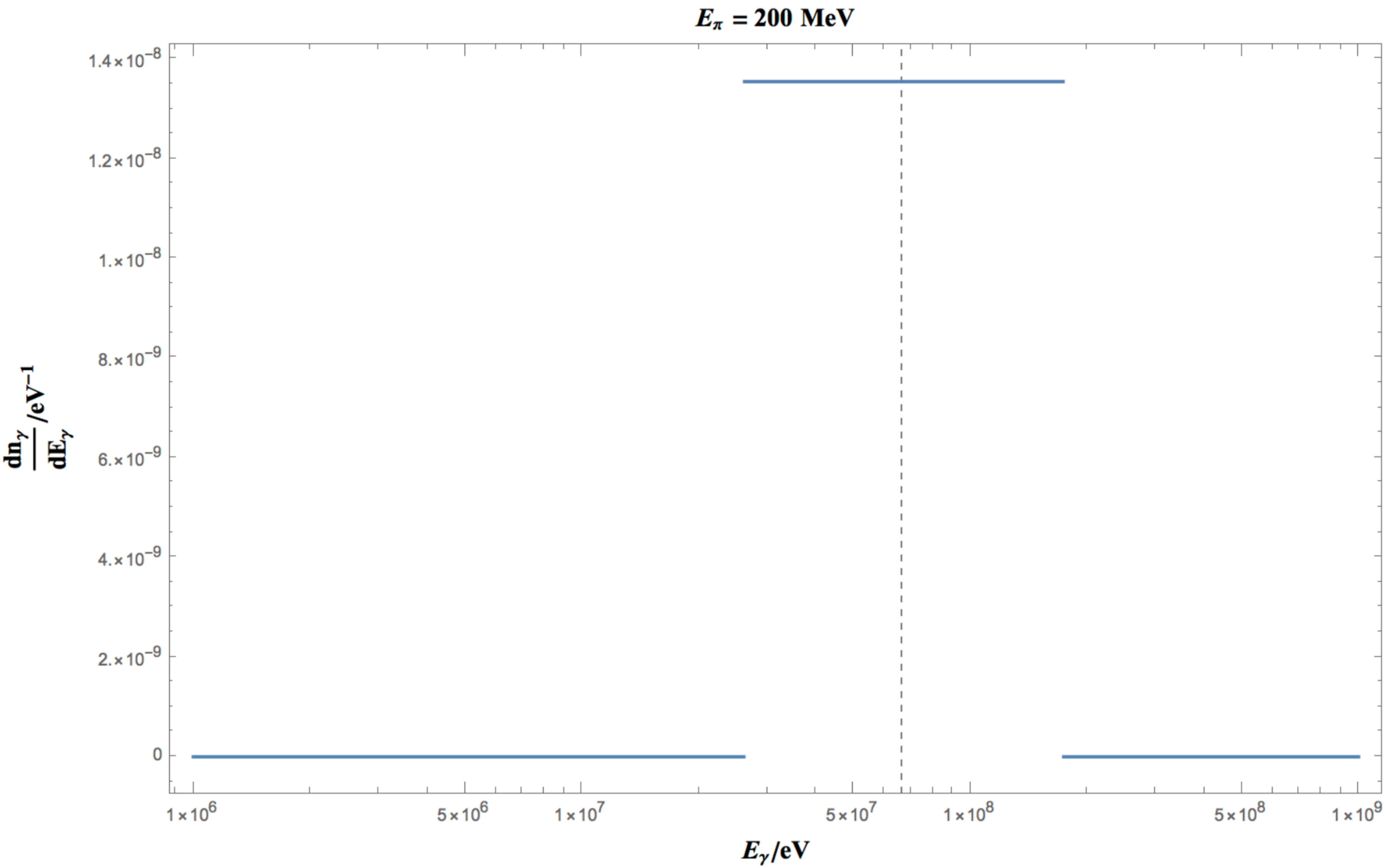
$$\sqrt{\frac{1-\beta_\pi}{1+\beta_\pi}}\frac{m_\pi}{2} \le E_\gamma^{(lab)} \le \sqrt{\frac{1+\beta_\pi}{1-\beta_\pi}}\frac{m_\pi}{2} .$$

- Also, given

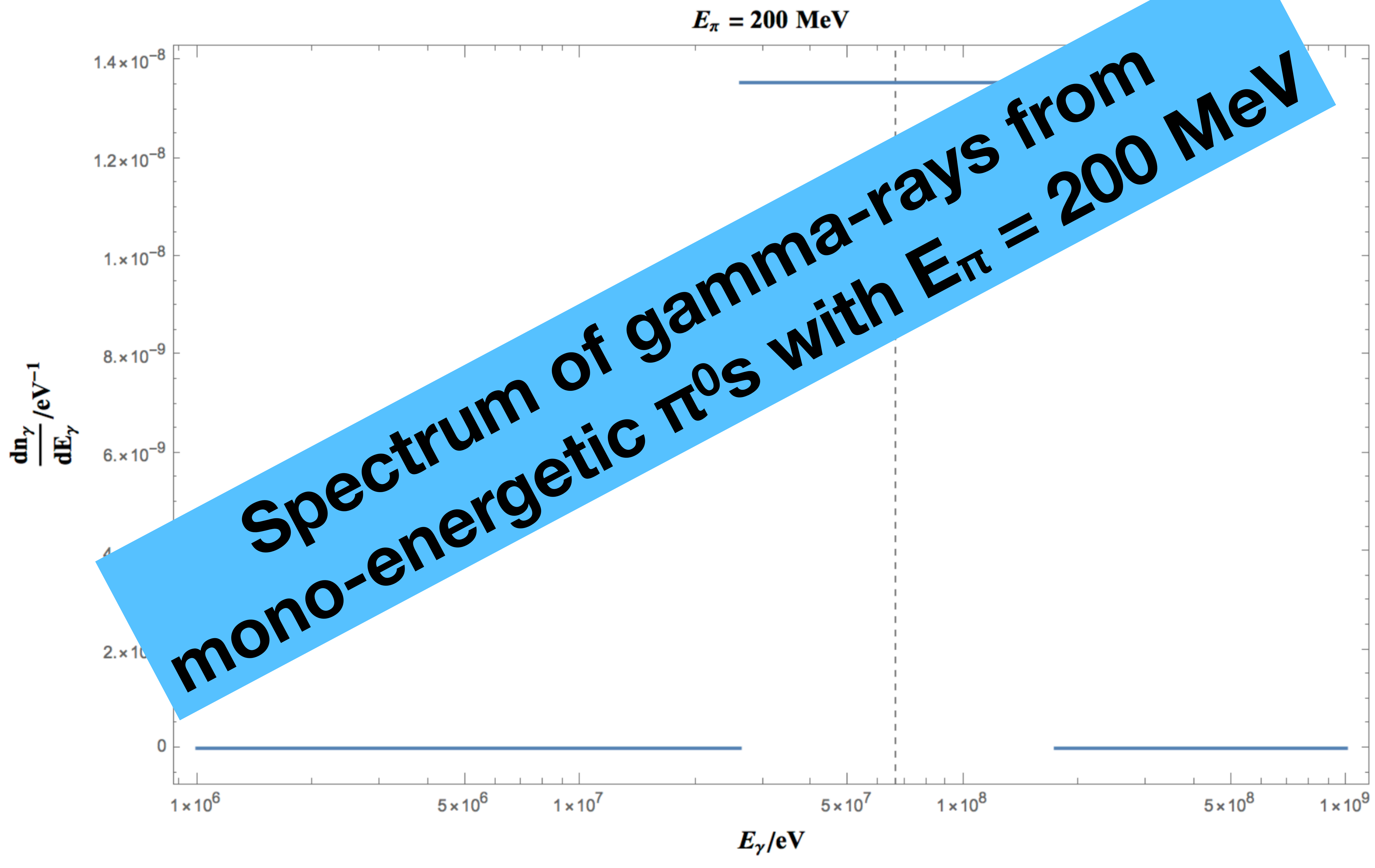
$$\frac{dn_\gamma}{dE_\gamma^{(lab)}} = const$$

there must be a ***flat distribution*** between these limits

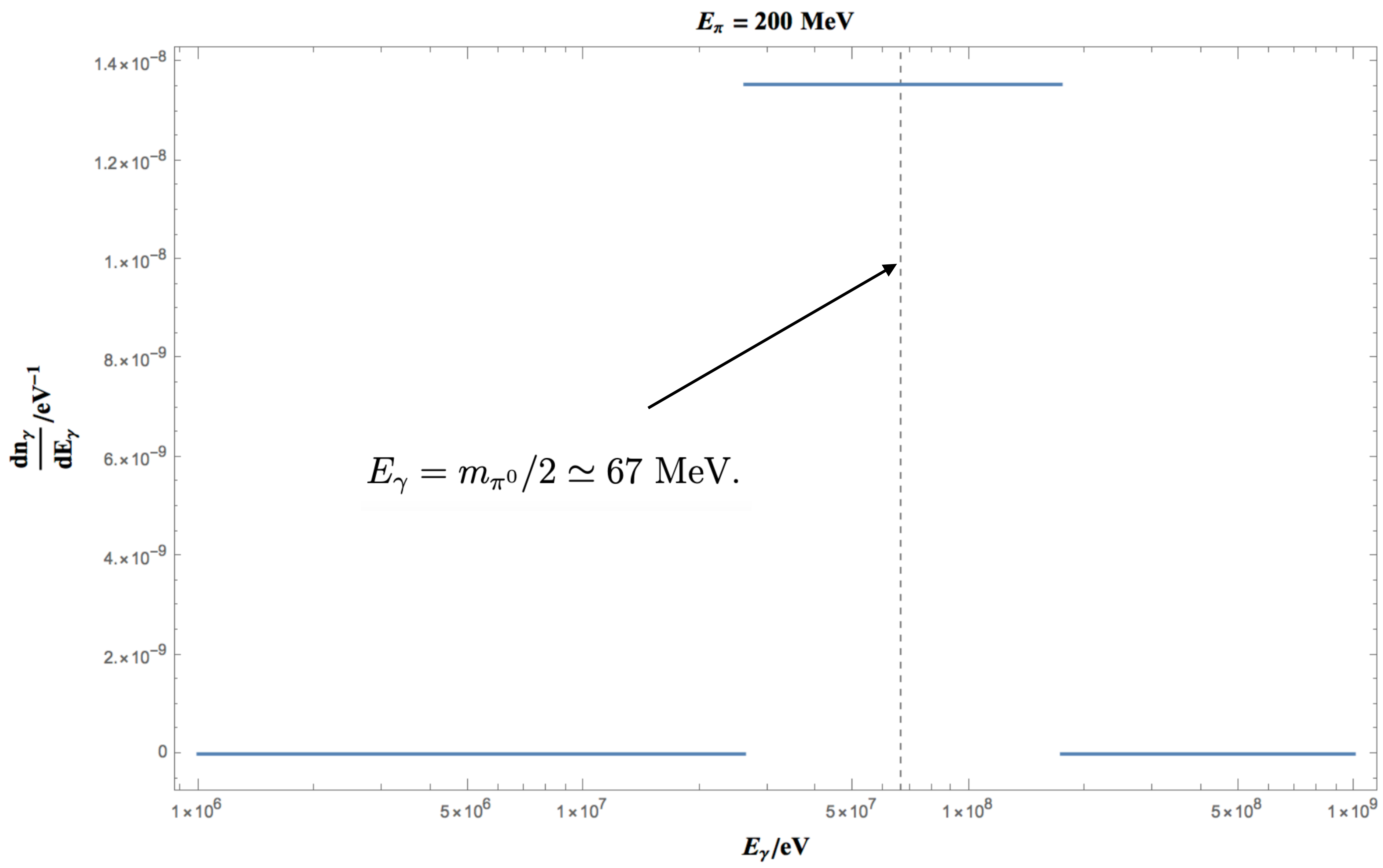
Spectrum of hadronic gamma-rays



Spectrum of hadronic gamma-rays



Spectrum of hadronic gamma-rays



Spectrum of hadronic gamma-rays

- Note that the geometric mean of $E_{\gamma,min}^{(lab)}$ and $E_{\gamma,max}^{(lab)}$ is:

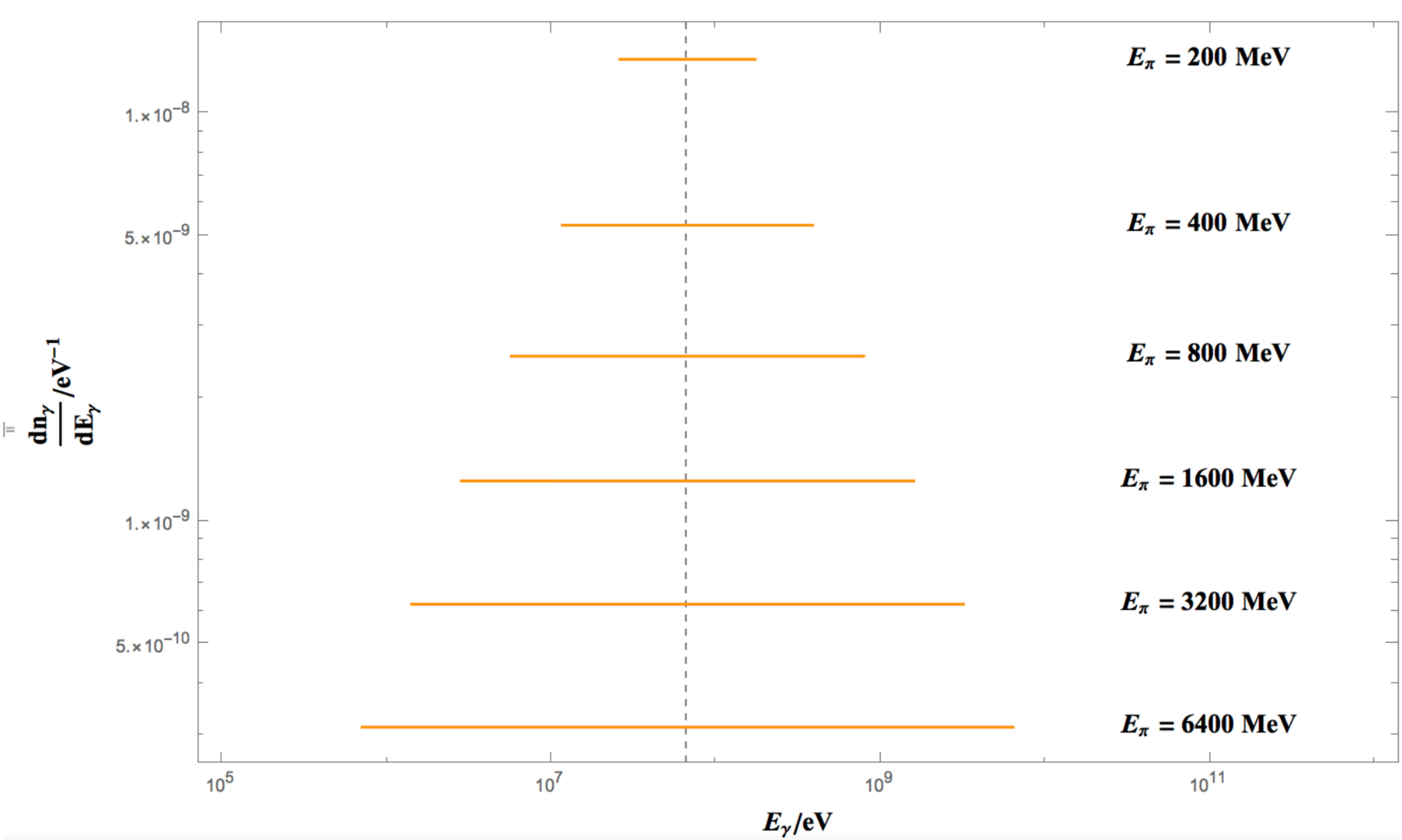
$$\langle E_\gamma^{(lab)} \rangle = \left[\sqrt{\left(\frac{1-\beta}{1+\beta}\right)\left(\frac{1+\beta}{1-\beta}\right)\left(\frac{m_\pi}{2}\right)^2} \right]^{1/2} = \frac{m_\pi}{2} \simeq 67\,\mathrm{MeV}$$

- Therefore, plotted on a log scale, $dn_\gamma/dE_\gamma^{(lab)}$ is symmetric around $m_\pi/2$

Spectrum of hadronic gamma-rays

- From a distribution of π^0s with different energies, we expect the resulting γ-ray distribution to be a weighted sum of “boxes” where the horizontal extent of each box increases with the energy of the parent π^0 responsible for the γ-rays populating that box

Spectrum of hadronic gamma-rays



Spectrum of hadronic gamma-rays

- From a distribution of π^0s with different energies, we expect the resulting γ-ray distribution to be a weighted sum of "boxes" where the horizontal extent of each box increases with the energy of the parent π^0 responsible for the γ-rays populating that box.
- This leads to a characteristic spectrum for hadronic γ-ray production that contains a characteristic π^0 decay bump at γ-ray energies in the vicinity of $m_{\pi^0} / 2$:

Spectrum of hadronic gamma-rays

- We have that

$$\frac{dP}{dE_\gamma}(E_\pi \to E_\gamma; E_\pi, E_\gamma) \equiv \frac{dn_\gamma}{dE_\gamma^{(lab)}} = \frac{2}{\sqrt{E_\pi^2 - m_\pi^2}}$$

- From which

$$\frac{dQ_\gamma(E_\gamma)}{dE_\gamma} = \int_{E_\gamma + \frac{m_\pi^2}{4E_\gamma}} dE_\pi \frac{2}{\sqrt{E_\pi^2 - m_\pi^2}} \frac{dQ_\pi(E_\pi)}{dE_\pi}$$

which for high energies ($E_\pi \gg m_\pi$) becomes:

$$\frac{dQ_\gamma(E_\gamma)}{dE_\gamma} \simeq \int_{E_\gamma} dE_\pi \frac{2}{E_\pi} \frac{dQ_\pi(E_\pi)}{dE_\pi}$$

Spectrum of hadronic gamma-rays

- Now take it that the π distribution is given by a power law: $$\frac{dQ_\pi}{dE_\pi}(E_\pi) = \frac{dQ_\pi}{dE_\pi}(E_\pi^0)\left(\frac{E_\pi}{E_\pi^0}\right)^\gamma$$
- Then we will have:

$$\frac{dQ_\gamma(E_\gamma)}{dE_\gamma} \simeq 2\frac{dQ_\pi}{dE_\pi}(E_\pi^0)\int_{E_\gamma} dE_\pi \frac{1}{E_\pi}\left(\frac{E_\pi}{E_\pi^0}\right)^\gamma = \frac{2}{(-\gamma)}\frac{dQ_\pi}{dE_\pi}(E_\gamma)$$

- Here γ is the 'spectral index', a number (and, in this context, does NOT mean photon)

Pion production in pp collisions

- In the astrophysical situation we are interested in, π's will be produced by the collisions of cosmic ray hadrons (p's and heavier ions) with ambient matter of density nH. For simplicity we continue to investigate the simplest case of pp collisions.
- The emissivity of (neutral) pions given a population of protons is given by:

$$\frac{dQ_\pi(E_\pi)}{dE_\pi} = \int_{E_p^{min}(E_\pi)} dE_p \frac{d\sigma_{pp}(E_p, E_\pi)}{dE_\pi} c\beta(E_p) n_H \frac{dn_p(E_p)}{dE_p}$$

where $E_p^{(min)}(E_\pi)$ is the minimum p energy required to produce a π of energy E_π.

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Differential spectrum 'beam' protons

where $E_p^{(min)}(E_\pi)$ is the minimum p energy required to produce a π of energy E_π.

Pion production in pp collisions

- From particle physics experiments, it can be shown that the high energy behaviour of $d\sigma_{pp}(E_p, E_\pi)/dE_\pi$ has a scaling form, i.e. that it depends only on the dimensionless Feynman x parameter which is given by the ratio of the energy of the outgoing secondary to the incoming beam particle's energy.

$$x_{(\pi)} \equiv \frac{E_\pi}{E_p}$$

- For instance, in the case of relevance, π^0 production:
- …and we can write:

$$\frac{d\sigma_{pp}(E_p, E_\pi)}{dE_\pi} \simeq \frac{d\sigma_{pp}(x)}{dE_\pi} = \sigma_{pp}^{inelas} \frac{f(x)}{E_\pi},$$

where $\sigma_{pp}^{inelas} \simeq const = 4 \times 10^{-26}\ \mathrm{cm}^2.$

Pion production in pp collisions

- The scaling function f(x) is obtained from collider data fitting.
- In the particular case of pp $\rightarrow \pi^0$'s, f(x) has the form:

$$f_{\pi^0}(x) = 2/3(1-x)^{7/2} + 1/2e^{-18x}$$

- You can think of the mean value of $f_{\pi 0}(x)$ over the x interval: $0 \rightarrow 1$, $\langle f_{\pi 0}(x) \rangle = 0.18$, as the **typical inelasticity** for the pp $\rightarrow \pi^0$ inelastic cross-section.

Pion production in pp collisions

- We may write: $$\frac{dQ_\pi(E_\pi)}{dE_\pi} \simeq c n_H \sigma_{pp}^{inelas} \int_{E_p^{min}(E_\pi)} dE_p \frac{1}{E_\pi} f_{\pi^0}\left(\frac{E_\pi}{E_p}\right) \frac{dn_p(E_p)}{dE_p}$$
- Now, we again assume a power law distribution for the input (beam) protons: $$\frac{dN_p}{dE_p}(E_p) = \frac{dN_p}{dE_p}(E_p^0)\left(\frac{E_p}{E_p^0}\right)^\gamma$$
- If we also assume the relativistic limit, we get the spectrum of pions they produce: $$\frac{dQ_\pi(E_\pi)}{dE_\pi} \simeq c n_H \sigma_{pp}^{inelas} \frac{dn_p}{dE_p}(E_\pi) \int_{E_\pi} dE_p \frac{1}{E_\pi} f_{\pi^0}\left(\frac{E_\pi}{E_p}\right)\left(\frac{E_p}{E_\pi}\right)^\gamma$$

Spectrum of hadronic gamma-rays

- Change variables to x = E_π/E_p:

$$\frac{dQ_\pi(E_\pi)}{dE_\pi} \simeq c n_H \sigma_{pp}^{inelas} \frac{dn_p}{dE_p}(E_\pi)\Lambda_{\pi^0}(\gamma)$$

- Where we have used the **spectrum weighted moment** (SWM) for pp → π^0**:**

$$\Lambda_{\pi^0}(\gamma) \equiv \int_0^1 dx x^{-(\gamma+2)} f_{\pi^0}(x)$$

- We can now combine pion decay to gamma-rays and pion production via pp collisions as:

$$\begin{aligned}\frac{dQ_\gamma(E_\gamma)}{dE_\gamma} &\simeq \frac{dn_p}{dE_p}(E_\gamma) c n_H \sigma_{pp}^{inelas} \frac{2}{(-\gamma)}\Lambda_{\pi^0}(\gamma) \\ &\equiv \frac{dn_p}{dE_p}(E_\gamma) c n_H \sigma_{pp}^{inelas} \Lambda_\gamma(\gamma)\end{aligned}$$

Spectrum of hadronic gamma-rays

- Above, the last line follows by the definition of the SWM for γ production

  demanding that
$$\Lambda_\gamma(\gamma) \equiv \frac{2}{(-\gamma)} \Lambda_{\pi^0}(\gamma) \,.$$

- This implies:
$$\frac{dQ_\gamma}{dE_\gamma}(E_\gamma) \equiv \frac{2}{(-\gamma)} \frac{dQ_\pi}{dE_\pi}(E_\gamma) \,.$$

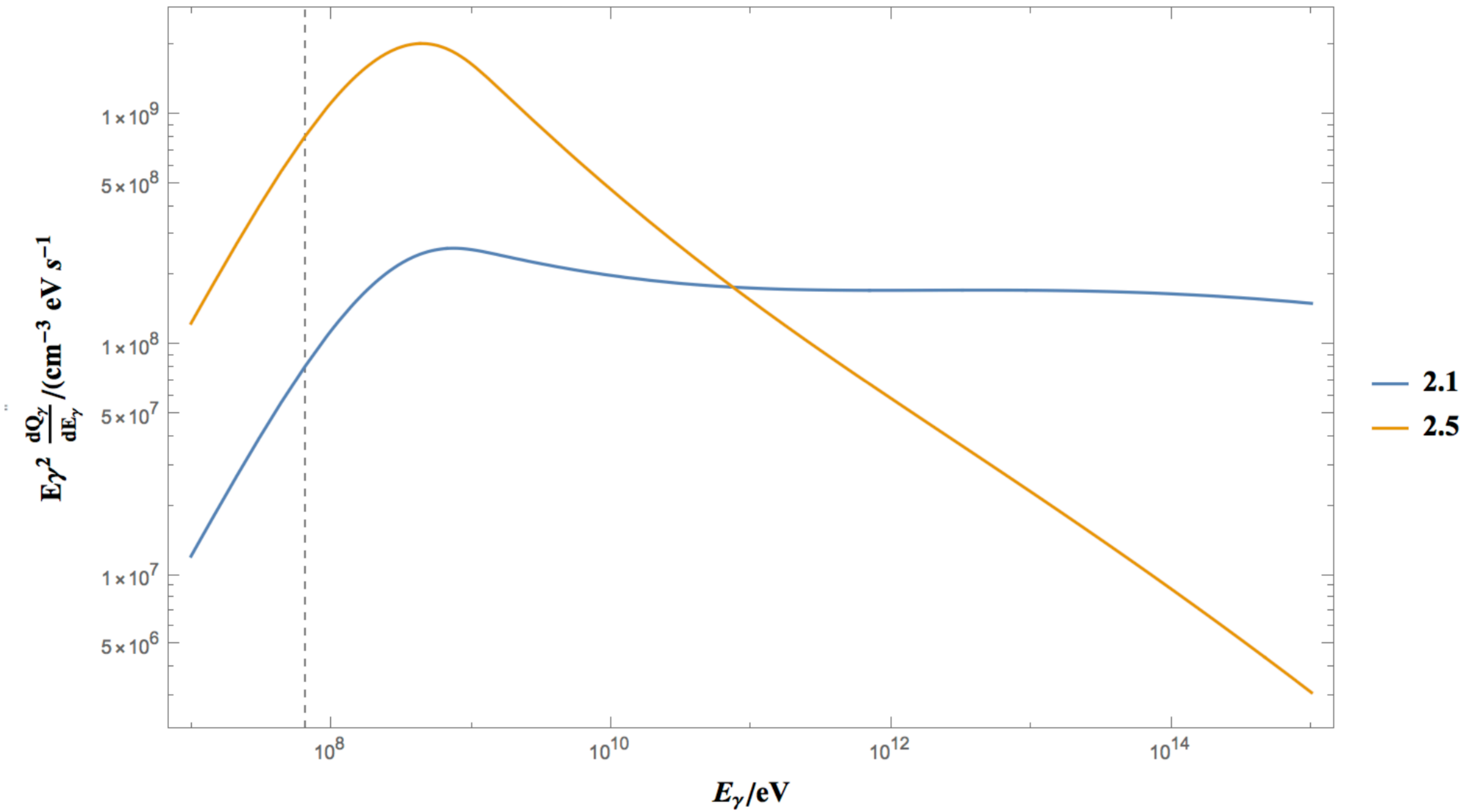
- Overall, importantly, **if the input p distribution is a power law of spectral index γ then, at high energies, the output γ distribution is also a power law with spectral index γ.**

- As a rule of thumb, at such energies – where scaling is a reasonable approximation – the mean energy of the parent proton of a given γ-ray of E_γ is $E_p \sim 10\ E_\gamma$

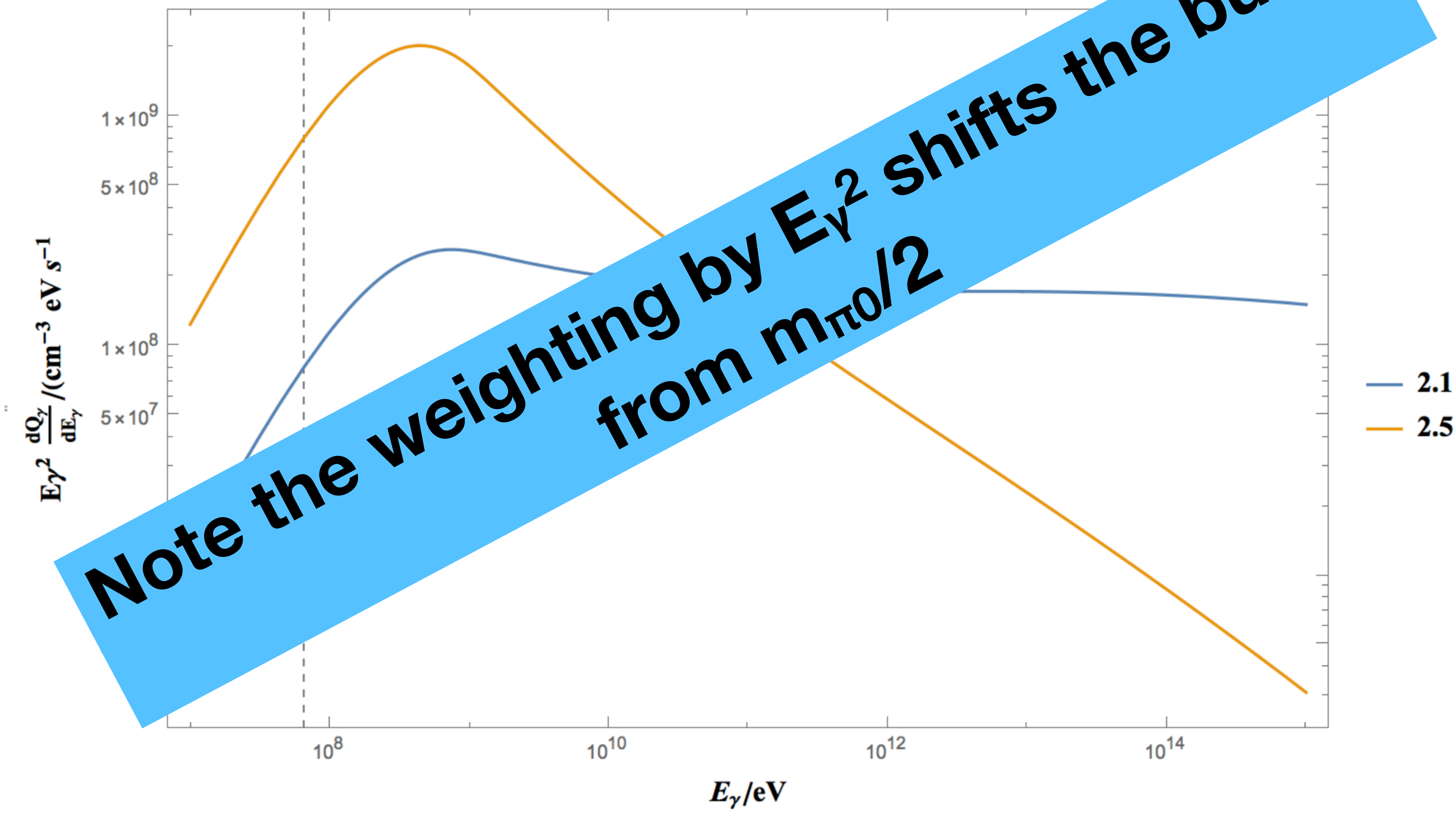
Phenomenology of hadronic gamma-rays

- If we can unambiguously find the π^0 decay bump in the γ-ray spectrum of a particular astro- physical object/region this has the implication that cosmic-ray (hadronic) collisions are taking place in that object/region

Phenomenology of hadronic gamma-rays



Phenomenology of hadronic gamma-rays



Note the weighting by $E_\gamma{}^2$ shifts the bump from $m_{\pi 0}/2$

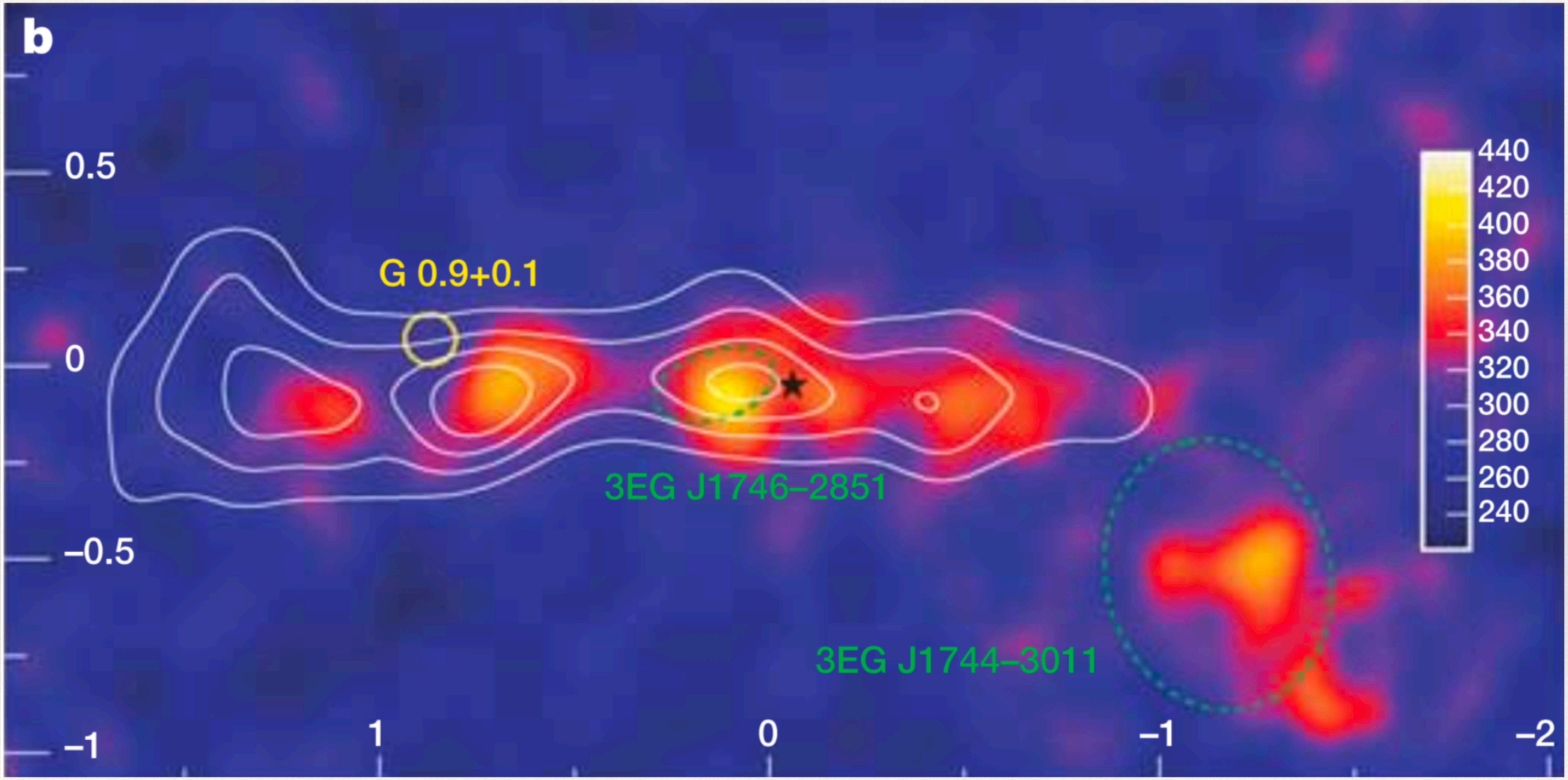
Phenomenology of hadronic gamma-rays

- If we can unambiguously find the π^0 decay bump in the γ-ray spectrum of a particular astro- physical object/region this has the implication that cosmic-ray (hadronic) collisions are taking place in that object/region
- Unfortunately, other leptonic emission processes (IC, bremsstrahlung) can confuse or even swamp this potential signal.
- Indeed, even were there no cosmic ray electrons present in the source, secondary electrons (i.e., both electrons and positrons created in the decay chain of $\pi^{\pm}$ also created via the pp collisions) produce an irreducible background to the π^0 decay signal which tends to wash out the bump.
- One place, however, where the spectral evidence for hadronic γ-ray production is unambiguous is for the Galactic plane. Indeed, this was established by EGRET, the predecessor instrument to the Fermi-LAT.

Phenomenology of hadronic gamma-rays

- Another signature of hadronic gamma-ray production is that is should correlate with ‘target’ gas density.

Phenomenology of hadronic gamma-rays



Phenomenology of hadronic gamma-rays

- In the previous figure: colours show the distribution of ~ TeV γ-rays in the Central Molecular Zone of the Milky Way, the central few 100 pc around the Galaxy's supermassive black hole (credit: Aharonian et al. 2006, Nature).
- The white contours show the column of molecular hydrogen as traced by molecular line emission from the CS molecule.
- There is a spatial correlation between the peaks of the molecular gas column and the γ-ray surface brightness indicating hadronic emission as the likely origin of the γ-rays.