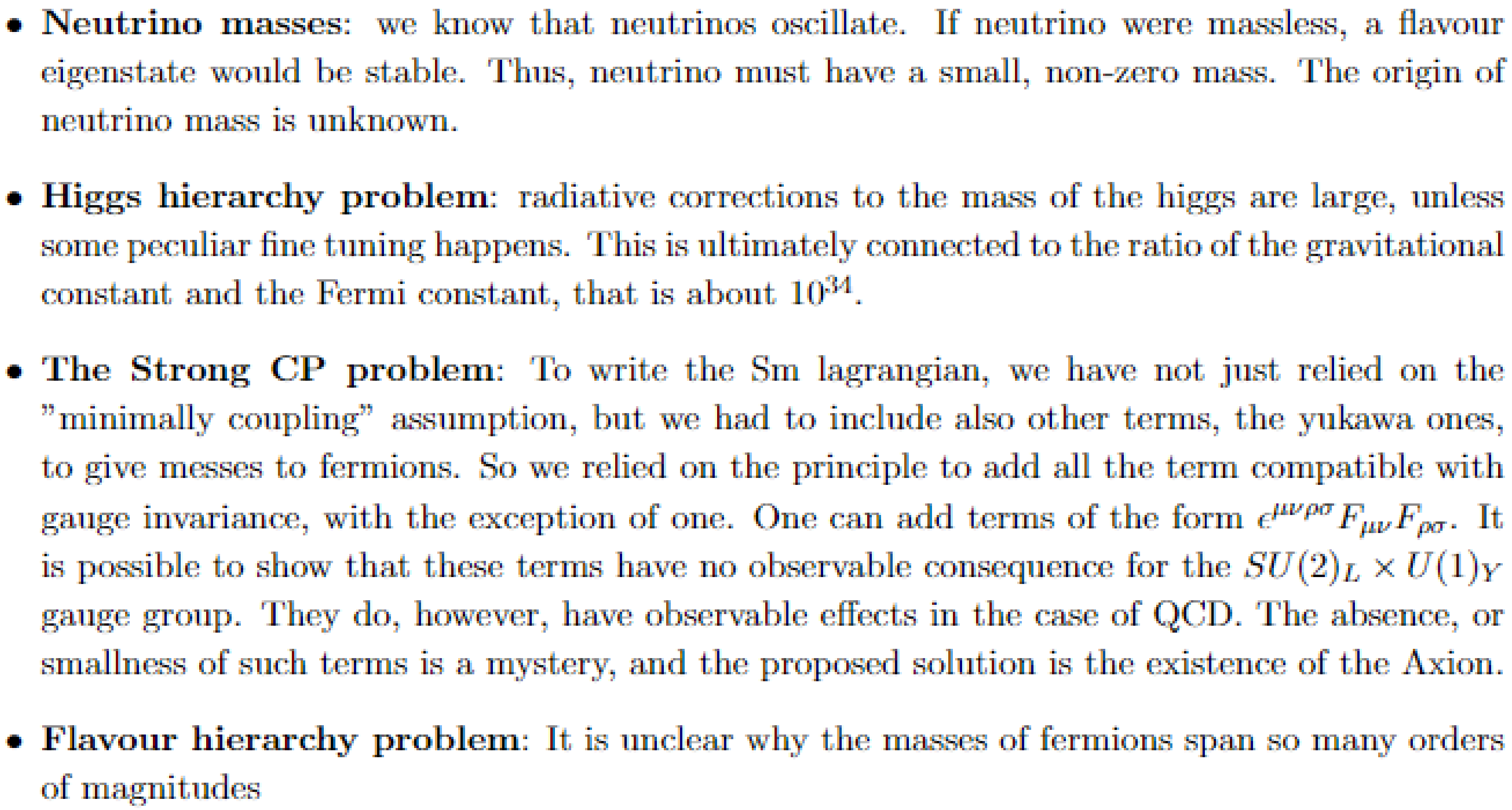
The Standard Model of Particle Physics

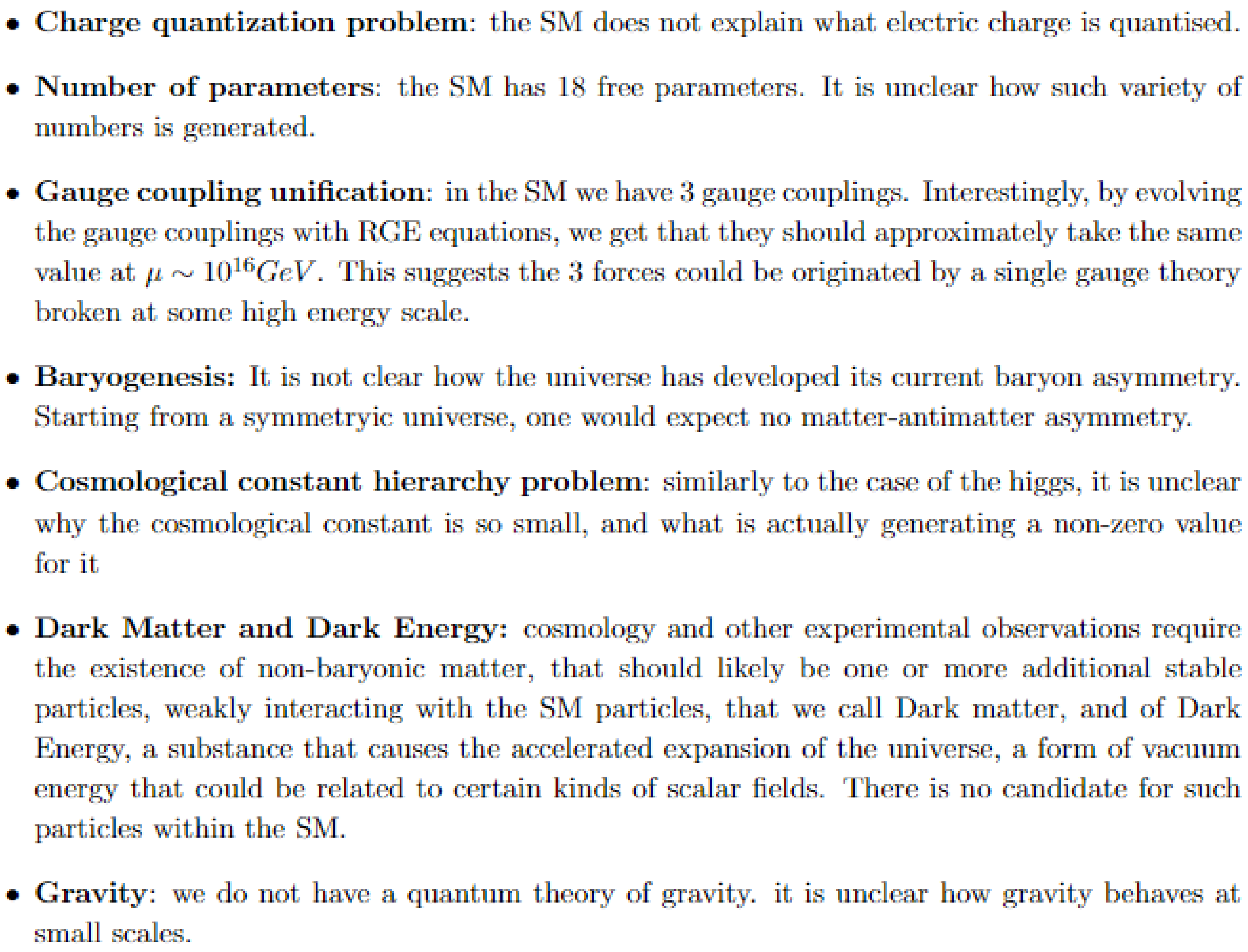
Lecture 6

Reasons to need to go beyond the SM



- **Neutrino masses**: we know that neutrinos oscillate. If neutrino were massless, a flavour eigenstate would be stable. Thus, neutrino must have a small, non-zero mass. The origin of neutrino mass is unknown.
- **Higgs hierarchy problem**: radiative corrections to the mass of the higgs are large, unless some peculiar fine tuning happens. This is ultimately connected to the ratio of the gravitational constant and the Fermi constant, that is about 10^{34}.
- **The Strong CP problem**: To write the Sm lagrangian, we have not just relied on the ”minimally coupling” assumption, but we had to include also other terms, the yukawa ones, to give messes to fermions. So we relied on the principle to add all the term compatible with gauge invariance, with the exception of one. One can add terms of the form $\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$. It is possible to show that these terms have no observable consequence for the $SU(2)_L \times U(1)_Y$ gauge group. They do, however, have observable effects in the case of QCD. The absence, or smallness of such terms is a mystery, and the proposed solution is the existence of the Axion.
- **Flavour hierarchy problem**: It is unclear why the masses of fermions span so many orders of magnitudes

Reasons to need to go beyond the SM



- **Charge quantization problem**: the SM does not explain what electric charge is quantised.
- **Number of parameters**: the SM has 18 free parameters. It is unclear how such variety of numbers is generated.
- **Gauge coupling unification**: in the SM we have 3 gauge couplings. Interestingly, by evolving the gauge couplings with RGE equations, we get that they should approximately take the same value at $\mu \sim 10^{16} GeV$. This suggests the 3 forces could be originated by a single gauge theory broken at some high energy scale.
- **Baryogenesis:** It is not clear how the universe has developed its current baryon asymmetry. Starting from a symmetryic universe, one would expect no matter-antimatter asymmetry.
- **Cosmological constant hierarchy problem**: similarly to the case of the higgs, it is unclear why the cosmological constant is so small, and what is actually generating a non-zero value for it
- **Dark Matter and Dark Energy:** cosmology and other experimental observations require the existence of non-baryonic matter, that should likely be one or more additional stable particles, weakly interacting with the SM particles, that we call Dark matter, and of Dark Energy, a substance that causes the accelerated expansion of the universe, a form of vacuum energy that could be related to certain kinds of scalar fields. There is no candidate for such particles within the SM.
- **Gravity**: we do not have a quantum theory of gravity. it is unclear how gravity behaves at small scales.

Neutrino masses

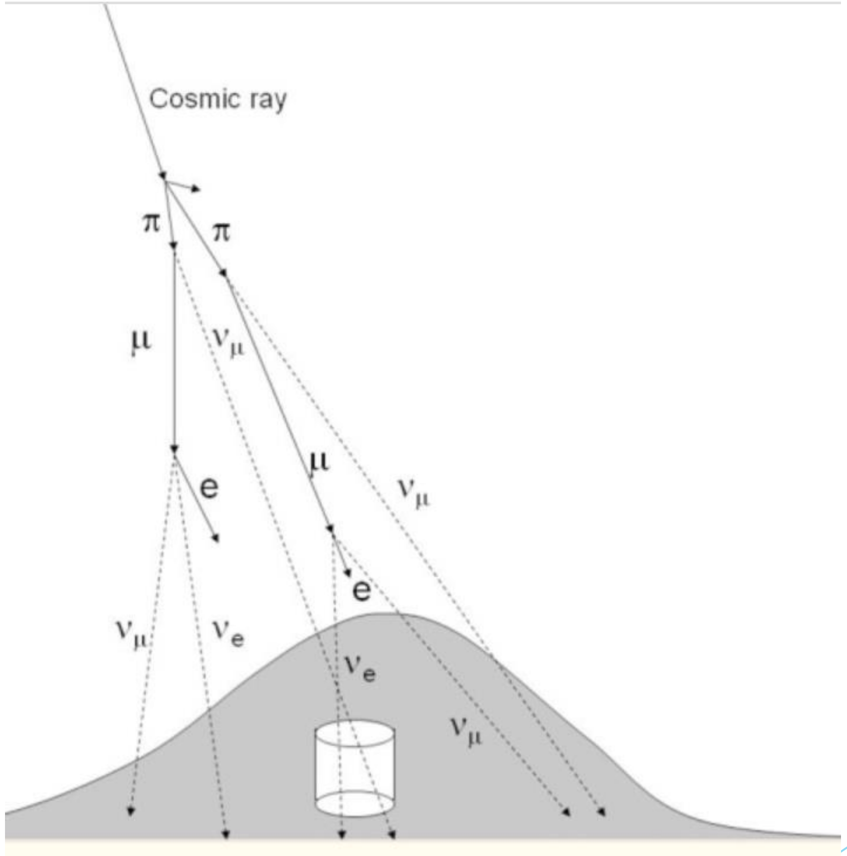
Cosmic rays (high energy protons mostly) can interact with upper layers of atmosphere and generate Pions. Pions decay chain produces $\nu_{e,\mu}$.

A prediction on the ratio of the 2 species of neutrino can be made.

Measuremenets confirmed the right amount of electron neutrino, but showed a deficit of muon neutrino, comparing to predictions.

Solar neutrinos also were showing a deficit: in the nuclear reactions in the Sun, only ν_e were expected to be produced.

However, experiments measuring ν_e only were measuring only 1/3 of the predicted neutrino flux (solar neutrino problem).



Neutrino masses

These mismatches were explained later on in terms of Neutrino oscillations. Evidence for oscillations in atmospheric neutrinos was announced in 1998. Evidence for oscillations in solar neutrinos, solving the solar neutrino problem, was announced in 2001.

Neutrino oscillations requires neutrino to have masses and mass differences, because massless particles are stable, i.e. do not oscillate.

In the SM, neutrinos have no right handed counterpart, and a Majorana mass term

$$\nu_L \nu_L^c$$

Is forbidden by Gauge invariance.

Just adding a right handed sterile neutrino does not solve the problem consistently with observations.

Neutrino masses: Seesaw mechanism

There are 3 types of Seesaw mechanisms, that can be used to give mass to neutrinos

They all rely on a common fact: by adding right handed neutrinos, one can have both a Dirac mass term, as usual, and a Majorana mass term for the right-handed fields, as they are gauge singlets. This would give rise to the following mass matrix

$$\begin{pmatrix} 0 & M \\ M & B \end{pmatrix}$$

Where M is the dirac mass term, coming from a mechanism similar to up, down quarks and charged leptons, therefore

$$M \approx O(10^{-6} - 1) \times v$$

On the other hand, the term B is the Majorana mass term for the right handed neutrinos, and theoretically it could lie at a very high energy scale. When $M \ll B$,

this matrix has 2 eigenvalues:

$$\lambda_+ \sim B, \lambda_- \sim -\frac{M^2}{B}$$

Thus one can have very heavy right handed sterile neutrinos, and very light left handed neutrinos.

The Seesaw scale B is thought to be close to the GUT scale (more in following slides).

Strong CP problem

Principle to build SM lagrangian:

- Make all kinetic terms invariant under gauge transformation
- Add all remaining terms compatible with gauge invariance

Note that such terms, even if not included in the theory at tree level, will be generated anyway by loop corrections, so you need to include them.

There are additional terms for the gauge bosons that are compatible with gauge symmetries:

$$\mathcal{L} = \frac{\theta_i g_i^2}{64\pi^2} \varepsilon^{\mu\nu\rho o} F^a_{\mu\nu} F^a_{\varrho o}$$

These terms violate P, but this is not a problem as P is already volated in the SM. These terms can be shown to be total derivatives for the $U(1)$ and the $SU(2)$ gauge fields, and therefore have no physical meaning.

The term for the $SU(3)$ gauge field, instead, has physical consequences, as it would originate an electric dipole moment for the neutron, that has been experimentally searched for and excluded at an impressive degree of accuracy!

Strong CP problem and the Axion

After diagonalizing thr Yukawa mass matrices, one gets that the angle θ_3 has become

$$\theta_3 \rightarrow \theta_3 - \arg\left(Det\ (Y^u Y^d)\right)$$

And one would expect $\theta_3 \sim O(1)$

Measurements from the electric dipole moment of the neutron impose

$$\theta_3 < 10^{-10}$$

The proposed solution for this problem is the one from Peccei-Quinn, the Axion. In the axion model, the lagrangian becomes

$$\mathcal{L} = \left(\theta_3 - \frac{a}{f}\right)\frac{g_i^2}{64\pi^2}\varepsilon^{\mu\nu\rho o} F^a_{\mu\nu} F^a_{\varrho o} + \cdots$$

Where a is the field of the pseudoscalar Axion, and we omitted other new terms related to it. This term now acts as an effective potential for the scalar axion field. We know that scalar particles develop a vev to sit at the minimum of the potential, in such case this is

$$< a > = \theta_3 f$$

So this term gets naturally canceled when rewriting the lagrangian in term of the true axion field with no vev.

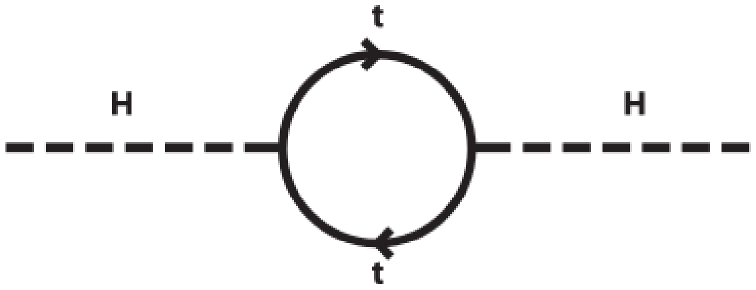
Hierarchy problem and Supersymmetry

In the SM, the SM higgs fields mass is not protected by any symmetry

The higgs field mass is affected by quadratic divergencies. Therefore, unless very finely tuned, one would expect ultimately it to lie at the Planck mass,

$$M_{pl} = 10^{18} GeV$$

Supersymmetry tries to solve this issue by postulating that every fermion has a supersymmetric scalar partner, and vice-versa. Scalar and fermion loop contributions have opposite signs, and so would cancel out identically.



Two higgs doublet models and extended scalar sectors

Its is worth mentioning that most of these ideas (Seesaw, PQ symmetry, Supersymemtry) can either be accomplished or require the existence of a more complicated scalar sector made, for example, of 2 higgs doublets.

The Lagrangian of a 2HDM is more complicated than the scalar one in the SM, but the essential ingredients are the same: EW symmetry breaking happens in the same way. Having 2 doublets means having:

2 scalars, 2 pseudoscalars and 2 charges scalars

Of these, 1 pseudoscalr and 1 charges scalar are “eaten” by the gauge bosons to acquire mass, and therefore one is left with

2 scalars, 1 pseudoscalar and 1 charges scalar.

These models are quite severely constrained by flavor physics, EW precision observables and higgs physics measurements. However, the 2HDM has the possibility to lie in the so called “decoupling limit”, where the second doublet is allowed to have any mass, regardless of any experimental constraint. The only problem with this is that it does not follow a naturalness principle.

Grand unification Theories

These theories aim to solve multiple issues at the same time.

In a GUT, the SM gauge group gets embedded in a larger simple gauge group.

Simple groups have the nice feature of having anomaly-free representations: thus one no more wonders where the intricate anomaly cancellation comes from.

The simples possible groups is $SU(5)$. Gauge bosons would belong the the adjoint representation, the 24. One needs also a scalar field in a 24 to break

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$$

Indeed one can check that after the breaking, the gauge bosons belong to the following representations

$$24 = (8{,}1{,}0) + (1{,}3{,}0) + (1{,}1{,}0) + \left(3{,}2{,} -\frac{5}{6}\right) + \left(\bar{3}, 2, \frac{5}{6}\right)$$

Grand unification Theories

These theories aim to solve multiple issues at the same time.

In a GUT, the SM gauge group gets embedded in a larger simple gauge group.

Simple groups have the nice feature of having anomaly-free representations: thus one no more wonders where the intricate anomaly cancellation comes from.

The simples possible groups is $SU(5)$. Gauge bosons would belong the the adjoint representation, the 24. One needs also a scalar field in a 24 to break

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

Indeed one can check that after the breaking, the gauge bosons belong to the following representations

$$24 = \boxed{(8,1,0)} + (1,3,0) + (1,1,0) + \left(3,2,-\frac{5}{6}\right) + \left(\bar{3},2,\frac{5}{6}\right)$$

Adjoint of SU(3): gluons

Grand unification Theories

These theories aim to solve multiple issues at the same time.

In a GUT, the SM gauge group gets embedded in a larger simple gauge group.

Simple groups have the nice feature of having anomaly-free representations: thus one no more wonders where the intricate anomaly cancellation comes from.

The simples possible groups is $SU(5)$. Gauge bosons would belong the the adjoint representation, the 24. One needs also a scalar field in a 24 to break

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$$

Indeed one can check that after the breaking, the gauge bosons belong to the following representations

$$24 = (8,1,0) + \boxed{(1,3,0)} + (1,1,0) + \left(3,2,-\frac{5}{6}\right) + \left(\bar{3},2,\frac{5}{6}\right)$$

Adjoint of SU(2): W fields

Grand unification Theories

These theories aim to solve multiple issues at the same time.

In a GUT, the SM gauge group gets embedded in a larger simple gauge group.

Simple groups have the nice feature of having anomaly-free representations: thus one no more wonders where the intricate anomaly cancellation comes from.

The simples possible groups is $SU(5)$. Gauge bosons would belong the the adjoint representation, the 24. One needs also a scalar field in a 24 to break

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

Indeed one can check that after the breaking, the gauge bosons belong to the following representations

$$24 = (8,1,0) + (1,3,0) + \boxed{(1,1,0)} + \left(3,2,-\frac{5}{6}\right) + \left(\bar{3},2,\frac{5}{6}\right)$$

B field

Grand unification Theories

These theories aim to solve multiple issues at the same time.

In a GUT, the SM gauge group gets embedded in a larger simple gauge group.

Simple groups have the nice feature of having anomaly-free representations: thus one no more wonders where the intricate anomaly cancellation comes from.

The simples possible groups is $SU(5)$. Gauge bosons would belong the the adjoint representation, the 24. One needs also a scalar field in a 24 to break

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

Indeed one can check that after the breaking, the gauge bosons belong to the following representations

$$24 = (8,1,0) + (1,3,0) + (1,1,0) + \boxed{\left(3,2,-\frac{5}{6}\right) + \left(\bar{3},2,\frac{5}{6}\right)}$$

New X,Y gauge bosons

Grand unification Theories: charge quant.

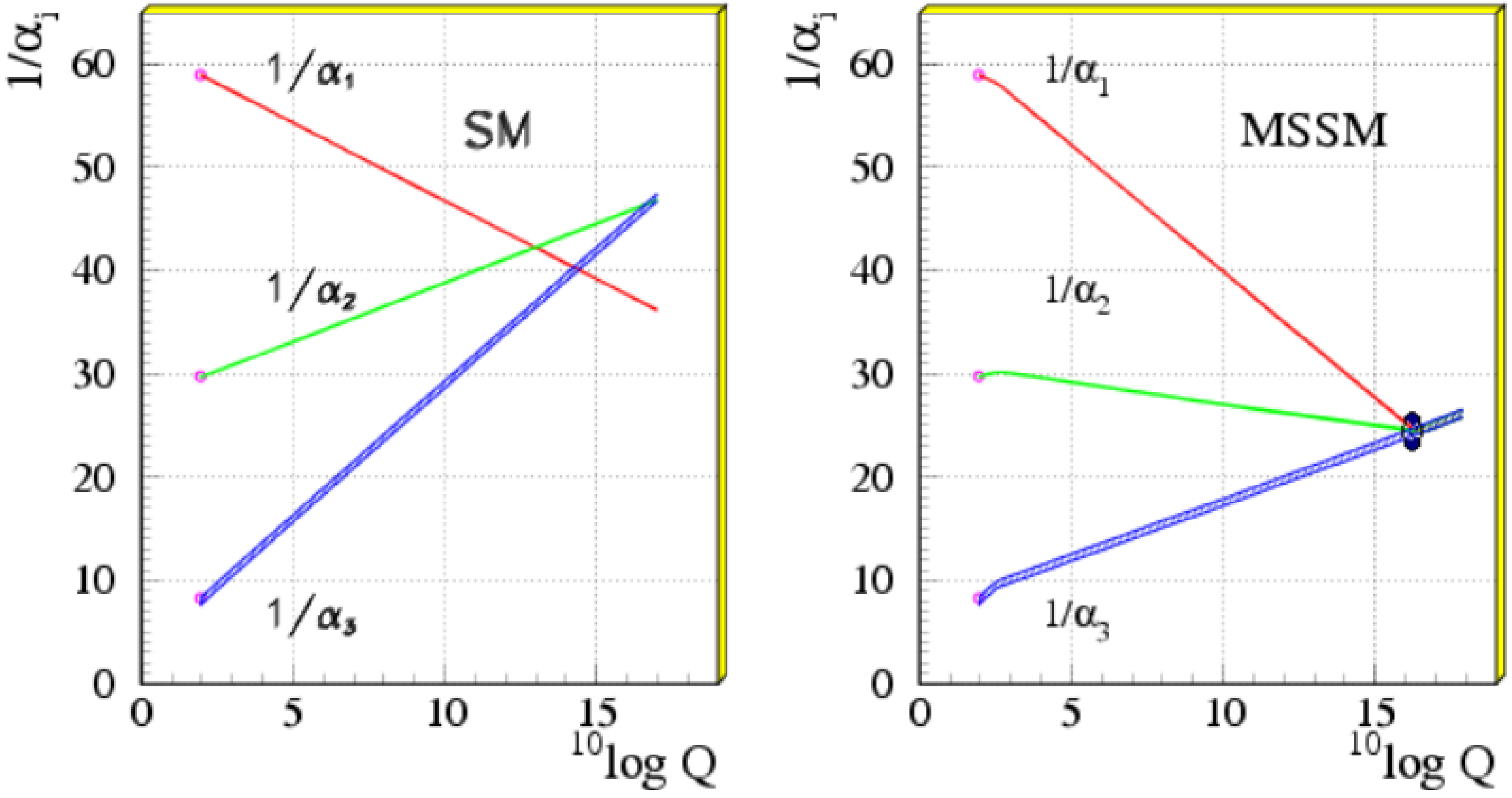
$$V_\mu = \begin{pmatrix} G^1_1 - \frac{2}{\sqrt{30}}B & G^1_2 & G^1_3 & \bar{X}^1 & \bar{Y}^1 \\ G^2_1 & G^2_2 - \frac{2}{\sqrt{30}}B & G^2_3 & \bar{X}^2 & \bar{Y}^2 \\ G^3_1 & G^3_2 & G^3_3 - \frac{2}{\sqrt{30}}B & \bar{X}^3 & \bar{Y}^3 \\ X^1 & X^2 & X^3 & \frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B & W^+ \\ Y^1 & Y^2 & Y^3 & W^- & -\frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B \end{pmatrix}$$

Gauge bosons in matrix representation.

The gauge boson B is associated with one of the diagonal generators of SU(5), as such, each particle in a representation has eigenvalues that have rational ratios.

This solves the charge quantization problem.

Grand unification Theories: gauge coupling unification



Gauge coupling unification + less parameters

Grand unification Theories

SM fermions can be accommodated in the 10 and $\bar{5}$ representations.

We need 2 additional higgs fields in the $5, \bar{5}$ representations to give masses to fermions, that generate a 2 hugs doublet model + 2 scalars charged under colour.

$$\bar{\mathbf{5}}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix} \qquad \mathbf{10}_F = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e_R \\ -d_1 & -d_2 & -d_3 & -e_R & 0 \end{pmatrix}$$

Grand unification Theories: Yukawa couplings

Comparing to the SM, we have less fields: in the SM we have 5 fermion fields, here we have only 2

There are only 2 possible yukawa terms:

$(\bar{5} + 10) + \bar{5}$ for down quarks end leptons

$(10 + 10) + 5$ for up quarks

This means having only 2 yukawa matrices

Again this should reduce the number of parameters and try to explain the origin of the fermion mass hierarchy

Unfortunately, it fails!

In particular, at the weak scale one obtains $\frac{m_s}{m_d} = \frac{m_\mu}{m_e}$ which is in serious disagreement with the data with $\frac{m_s}{m_d} \sim 20$ and $\frac{m_\mu}{m_e} \sim 200$.

Baryogenesis

Generation of matter-antimatter asymmetry requires Sakharov conditions:

1. Baryon number violation. SM allows it, non perturbatively. Remember that the global $U(1)_B$ symmetry is anomalous in the SM.
2. Breaking of C and CP symmetries. P and C are violated in the SM, and also CP, but the amount of CP violation in the SM is not sufficient to explain Baryogenesis.
3. Departure from thermal equilibrium. This could be accomplished by a first order EW phase transition. Unfortunately, the EW phase transition in the SM is not strongly first order.

Cosmological constant

It generates a sort of vacuum energy, however this energy is

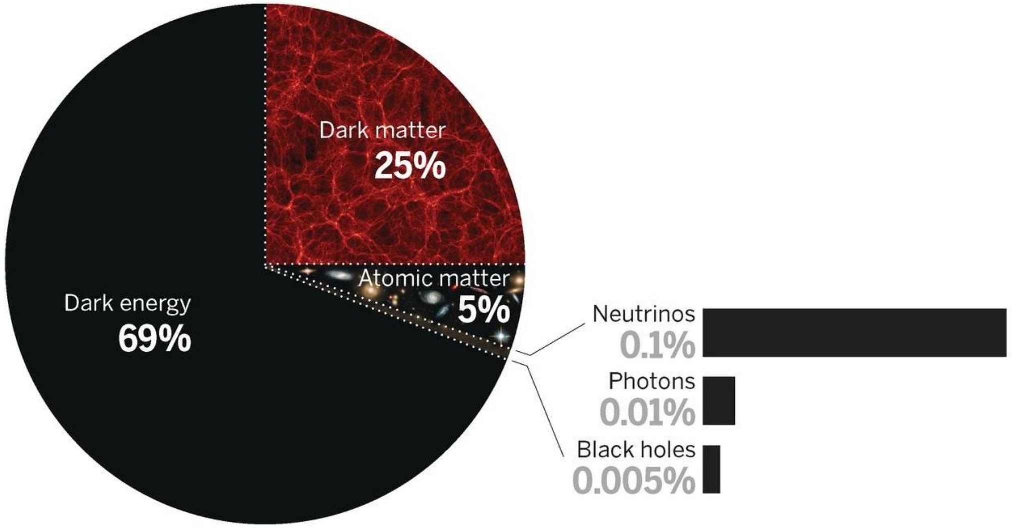
$$\rho_{vac} \approx (2.24 \times 10^{-3} eV)^4$$

While once again one would expect this to be set by the order of the Planck mass. The ratio of observed to expected is

$$\frac{\rho_{vac}}{M_{pl}^4} \approx \frac{(10^{-3} eV)^4}{(10^{18} GeV)^4} \sim 10^{-120}$$

Dark Matter and Dark energy

- Baryonic matter composes 5% of the energy content of the universe
- 25% should be composed by some form of matter that is non-baryonic, not interacting with light and “cold”, i.e. not moving at relativistic speeds
- While there are proposals for astrophysical object (PBH), the most accredited candidate is one or more stable BSM particles
- Finally, 70% of the energy should be made of dark energy, whose simplest explanation comes from the cosmological constant…



Gravity

Einstein theory of General relativity has undergone its last check by the detection of gravitational waves.

However, it is a classical theory, and we have been unable to find a way to quantize it. A quantum theory of gravity is very far from discovery at the moment.

Gravity should be mediated by gravitons, presumably massless particles, and its strength should become comparable to the other forces around the Planck scale.