The Standard Model of Particle Physics

Lecture 5

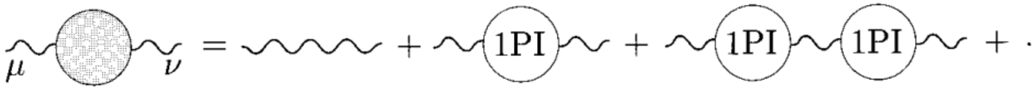
Loop Corrections to propagator

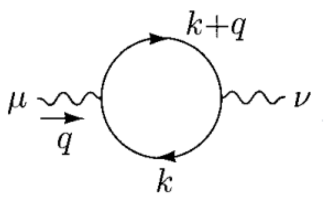
When considering loop diagrams, I can calculate diagrams that give corrections to propagators.

At some fixed order in perturbation theory, I need t calculate all 1 particle irreducible diagrams.

I can then find the "corrected" propagator by summing up an infinite series

$$D_F = D \sum (\Pi D)^n$$





Where Π is the result of the regularised 1 loop amplitude, with stripped external lines. We can find this easily for a scalar:

$$D_F = \frac{i}{q^2 - M^2} \sum (\frac{i}{q^2}(-i\Pi))^n = \frac{i}{q^2 - M^2} \sum (\frac{\Pi}{q^2 - M^2})^n$$

$$= \frac{i}{q^2 - M^2} \frac{1}{1 - \frac{\Pi}{q^2 - M^2}} = \frac{i}{q^2 - M^2 - \Pi}$$

Back to optical theorem

The optical theorem, for a one particle state, told us that

$$Im[\Pi] = M\Gamma$$

So we can write

$$\Pi = \delta M^2 + iM\Gamma$$

The propagator becomes

$$\frac{i}{q^2 - M^2 - (\delta M^2 + iM\Gamma\ )}$$

The real parts

$$M^2 + \delta M^2 = M'^2$$

Combine to give the renormalized mass, that substitutes the base mass M^2.

But the interesting thing is that an imaginary part appears if the particle is not stable ($\Gamma \neq 0$):

$$\frac{i}{q^2 - M'^2 - iM\Gamma}$$

Back to optical theorem

Starting back from the equation

$$2Im[M_{ii}] = \int |M_{ij}|^2 d\Phi_{LIPS} \tag{1.98}$$

We can specialise now to the case of a 2 particle initial and final state. For simplicity, we also assume particles of the same mass, but the result will be general

$$2Im[M_{ii}] = \frac{1}{8\pi}\sqrt{1-\frac{4m_j^2}{s}}|M_{ij}|^2 \tag{1.99}$$

We can now place an upper bound on the LHS and a lower bound on the RHS

$$2|M_{ii}| \geq 2Im[M_{ii}] = \sum_j \frac{1}{8\pi}\sqrt{1-\frac{4m_j^2}{s}}|M_{ij}|^2 \geq \frac{1}{8\pi}\sqrt{1-\frac{4m_i^2}{s}}|M_{ii}|^2 \tag{1.100}$$

We get the inequality

$$2|M_{ii}| \geq \frac{1}{8\pi}\sqrt{1-\frac{4m_i^2}{s}}|M_{ii}|^2 = \frac{v}{8\pi}|M_{ii}|^2 \tag{1.101}$$

$$|M_{ii}| \leq \frac{16\pi}{v} \tag{1.102}$$

We obtain a bound on the elastic cross section $i \to i$ (we use the c.o.m. frame)

$$\sigma = \frac{1}{2E_1 2E_2|v_1 - v_2|}|M_{ii}|^2\frac{v}{8\pi} \tag{1.103}$$

$$= \frac{1}{8E^2 v}|M_{ii}|^2\frac{v}{8\pi} \tag{1.104}$$

Back to optical theorem

$$\leq \frac{1}{64\pi E^2}\left(\frac{16\pi}{v}\right)^2 = \frac{4\pi}{E^2 v^2} = \frac{4\pi}{p_{com}^2} \to_{s\to\infty} \frac{16\pi}{s} \tag{1.105}$$

We can verify this bound using our scalar example, but this time let's assume that the particle S decays into 2 distinguishable particles f the same mass, so that we don't have the 1/2 factor in the phase space. The width of S is

$$\Gamma = \frac{\mu^2}{16\pi M}\sqrt{1 - \frac{4m^2}{M^2}} \tag{1.106}$$

The dressed propagator for S becomes

$$\frac{i}{Q^2 - M^2 - iM\Gamma} \tag{1.107}$$

The matrix element for $\eta\eta' \to \eta\eta'$ is

$$i\mathcal{M} = \frac{i(-i\mu)^2}{s - M^2 - iM\Gamma} \tag{1.108}$$

$$|\mathcal{M}|^2 = \frac{\mu^4}{(s - M^2)^2 + M^2\Gamma^2} \tag{1.109}$$

$$\sigma(s) = \frac{1}{2sv}\frac{v}{8\pi}|\mathcal{M}|^2 = \frac{1}{16\pi s}\frac{\mu^4}{(s - M^2)^2 + M^2\Gamma^2} \tag{1.110}$$

The peak of the cross section is reached at the resonance, for $s = M^2$. For such energy we get

$$\sigma(M^2) = \frac{\mu^4}{16\pi M^4\Gamma^2} = \frac{\mu^4}{16\pi M^4 \frac{\mu^4}{(16\pi)^2 M^2} v^2} \tag{1.111}$$

$$= \frac{1}{M^2 \frac{1}{(16\pi)} v^2} = \frac{16\pi}{sv^2} = \frac{4\pi}{p_{com}^2} \tag{1.112}$$

From the SM to the Fermi Theory: EFT

Starting from a renormalisible theory like the SM, one can get an Effective Field Theory, like the Fermi Theory of Weak interactions, by integrating out the heavy degrees of freedom, in our case the W boson. The relevant part of the SM lagrangian reads

$$-\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + M_W^2 W_\mu^+ W^{-\mu} + \frac{g}{\sqrt{2}}\left(\bar{Q}_L \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix} Q_L + \bar{L}_L \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix} L_L\right)$$

$$\rightarrow M_W^2 W_\mu^+ W^{-\mu} + \frac{g}{\sqrt{2}}\left(\bar{\nu}_L W^+ e_L + \bar{e}_L W^- \nu_L + \bar{u}_L W^+ V_{CKM} d_L + \bar{d}_L V_{CKM}^\dagger W^+ u_L\right)$$

Where we have removed the kinetic term. Without kinetic term, W is not anymore a propagating field, but just a classical one, and as such it can be removed by using Euler-Lagrange Equations:

$$\frac{\partial \mathcal{L}}{\partial W_\mu^+} = M_W^2 W^{-\mu} + \frac{g}{\sqrt{2}}\left(\bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu V_{CKM} d_L\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial W_\mu^-} = M_W^2 W^{+\mu} + \frac{g}{\sqrt{2}}\left(\bar{e}_L \gamma^\mu \nu_L + \bar{d}_L V_{CKM}^\dagger \gamma^\mu u_L\right) = 0$$

From the SM to the Fermi Theory: EFT

If we define

$$J^{+\mu} = \bar{\nu}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu V_{CKM} d_L$$
$$J^{-\mu} = \bar{e}_L \gamma^\mu \nu_L + \bar{d}_L V_{CKM}^\dagger \gamma^\mu u_L$$

If we solve the previous equations in terms of the fields $W^{\pm\mu}$, and substitute back into the lagrangian, we get

$$\mathcal{L} = \frac{g^2}{2M_W^2} J_\mu^- J^{+\mu} - \frac{g^2}{2M_W^2} J_\mu^- J^{+\mu} - \frac{g^2}{2M_W^2} J_\mu^- J^{+\mu} = -\frac{g^2}{2M_W^2} J_\mu^- J^{+\mu} = -\frac{4G_F^2}{\sqrt{2}} J_\mu^- J^{+\mu}$$

This is the original lagrangian proposed by Fermi to describe the physics of the weak interactions.

Muon Decay $\mu^- \to e^-\nu_\mu\bar{\nu}_e$

Leaving out neutrinos for now, the lightest non-massless particle is the electron, that is stable.

The next particle going up in mass is the muon. Being heavier than the electron, and as there is no symmetry forbidding it, the muon can decay. If we write the decay amplitude using the Fermi theory we get

$$-\frac{4G_F^2}{\sqrt{2}}\bar{v}(\nu_\mu)\gamma^\mu\frac{1-\gamma^5}{2}u(\mu^-)\bar{u}(e^-)\gamma_\mu\frac{1-\gamma^5}{2}v(\bar{\nu}_e)$$

If we write it using the SM lagrangian, in the unitary gauge, we get

$$\frac{g^2}{2}\bar{v}(\nu_\mu)\gamma_\mu\frac{1-\gamma^5}{2}u(\mu^-)\bar{u}(e^-)\gamma_\nu\frac{1-\gamma^5}{2}v(\bar{\nu}_e)\frac{g^{\mu\nu}-\frac{q^\mu q^\nu}{M_W^2}}{q^2-M_W^2}$$

If we neglect the q^μ terms, that are suppressed by $\frac{m_f}{M_W}$, and neglect $q^2 \ll M_W^2$ in the denominator, we get

$$-\frac{g^2}{2M_W^2}\bar{v}(\nu_\mu)\gamma^\mu\frac{1-\gamma^5}{2}u(\mu^-)\bar{u}(e^-)\gamma_\nu\frac{1-\gamma^5}{2}v(\bar{\nu}_e)$$

That is exactly the same result as in Fermi Theory.

Pions decay

Pions have the following valence quarks:

$$\pi^+ = u\bar{d}, \pi^- = d\bar{u}, \pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$

Through what force can they decay?

$SU(2)$ isospin symmetry means all these particle behave identically under the strong force, and as they are the lightest coloured particle they cannot decay strong.

Remember that the pions are the pseudo-goldstone bosons associated to the symmetry $SU(2)_A$, the conserved currents being

$$\bar{Q}\gamma^\mu t^a Q$$

What about electromagnetic interaction? Extending the QCD lagrangian with QED, it breaks the isospin symmetry, as u and d have different charge. However QED conserved the number of u quarks and d quarks separately, and so cannot make charged pions decay.

However, QED explains the $\pi^+ - \pi^0$ mass difference, and can make the π^0 decay through the anomaly, $\pi^0 \to \gamma\gamma$.

π^+ can therefore only decay by weak interaction $\pi^+ \to \mu^+\bar{\nu}_\mu$.

Kaon decay

Next particle in order of mass is the charged Kaon K^+.

$$m(K^+) = 493MeV, m(\pi^+) = 139MeV$$

Similarly to the pion, cannot decay strong or em.

Possible weak decays:

$$K^+ \to \mu^+ \bar{\nu}_\mu$$
$$K^+ \to \pi^+ \pi^0$$
$$K^+ \to \pi^+ \pi^0 \pi^0, \pi^+ \pi^+ \pi^-$$

We already talked about the $K_{L,S}$ decays

Tau decay

Finally, we can consider the tau decay. We use this to show the power of EFT combined with dimensional analysis.

For the muon, there is only one decay channel $\mu^- \to e^- \nu_\mu \bar{\nu}_e$

Neglecting the electro mass, the amplitude for this process should be

$$\mathcal{M} = A G_F m_\mu^2$$

Where A is a dimensionless constant, as $\mathcal{M}$ should be dimensionless

Therefore the decay amplitude will be

$$\Gamma = \frac{1}{m_\mu} A' G_F^2 m_\mu^4 m_\mu^2$$

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Initial State normalization

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Squared amplitude

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Phase space

Tau decay: leptonic modes

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Neglecting the electro mass, the amplitude for this process should be

$$\mathcal{M} = AG_F m_\mu^2$$

Where A is a dimensionless constant, as $\mathcal{M}$ should be dimensionless

Therefore the decay amplitude will be

$$\Gamma_\mu = \frac{A'}{m_\mu} G_F^2 m_\mu^4 m_\mu^2 = A' G_F^2 m_\mu^5$$

We can use this to predict

$$\Gamma_{\tau \to l} = \Gamma_\mu \left(\frac{m_\tau}{m_\mu} \right)^5, l = e, \mu$$

Tau decay: hadronic modes

The τ mass is large enough that allows decay into mesons. As a simple way to get a rough estimate, we can consider as a final state the state $d\bar{u}$. This will bring a Feynman diagram and an amplitude of the same for as for the leptonic decay modes.

BUT! Final state quarks have 3 colors. When summing up the quared amplitude on final states, therefore, you will get an additional factor of 3

We can use this to predict

$$\Gamma_{\tau \to hadr} = 3\Gamma_\mu \left(\frac{m_\tau}{m_\mu}\right)^5$$

Summing up all the decay widths we get

$$\Gamma_\tau = 5\Gamma_\mu \left(\frac{m_\tau}{m_\mu}\right)^5$$

That is a good estimate.

W decay and production

W mass is 80 GeV. This allows it to decay into fermion pairs, except bt. The W has also direct couplings to fermion pairs.

At collider we can collide p, e. To produce a W we need either to smash $e\nu$ or 2 quarks. As we cannot accelerate neutrinos, we need an hadron accelerator.

To detect the W one needs to select some decay mode.

Decays to quarks are more difficult to detect at an hadron collider, cause there is a lot of background of hadronic final states.

Decays to a charged lepton and a neutrino have the problem that the neutrino cannot be detected and therefore one cannot measure the invariant mass

$$(p_l + p_\nu)^2 = M_W^2$$

The leptonic decays were anyway the better ones, and the W was discovered in 1983.

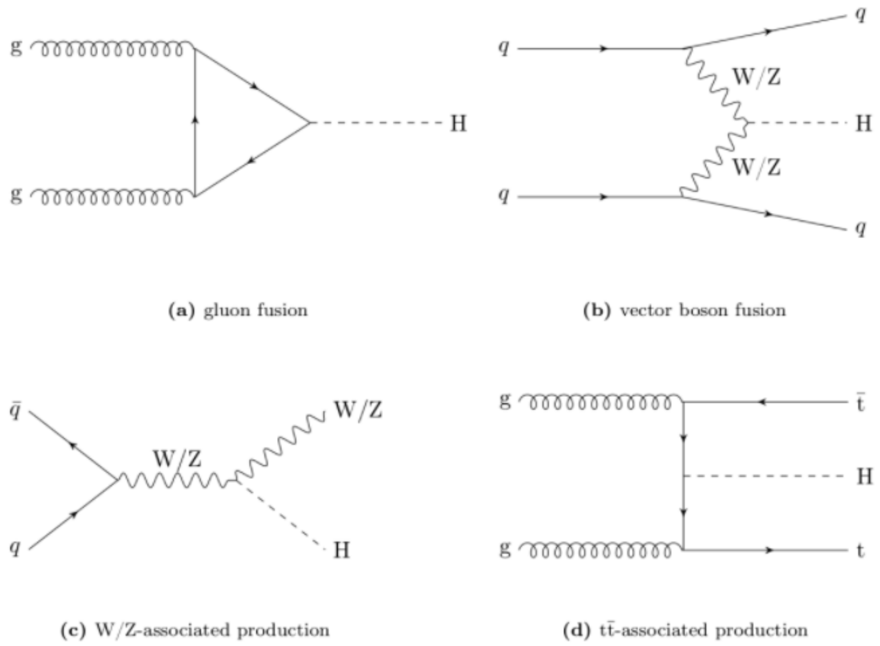
Z decay and production

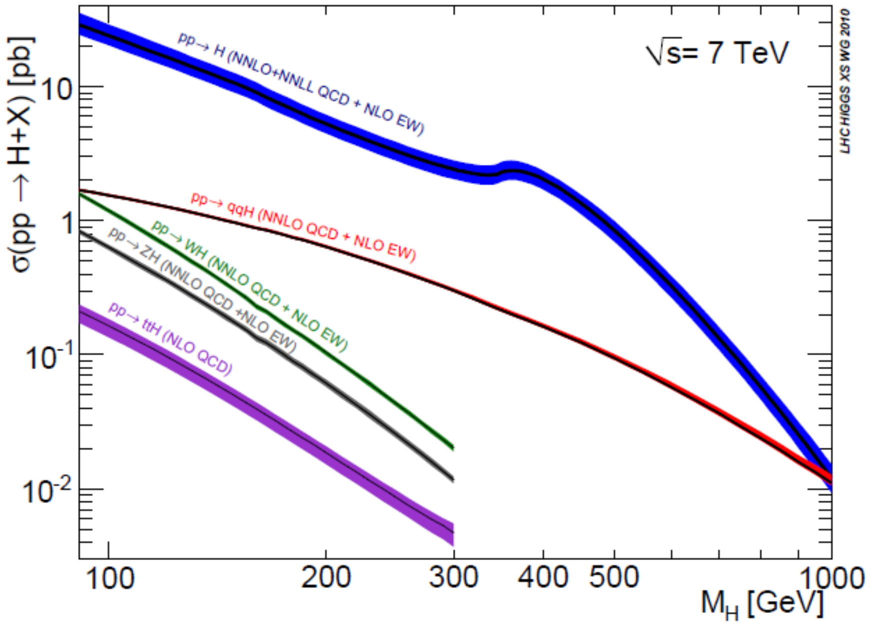
mass is 91 GeV. It couples to fermion antifermion pairs. Can be produced and decay using hadron collider and electron positron collider.

Similarly to the W, the Z was produced the first time at hadronic collider, and also in this case one needs to choose between hadronic and leptonic decay modes.

Similarly to the W, the leptonic decays are more “clean” channels to be detected.

Higgs production

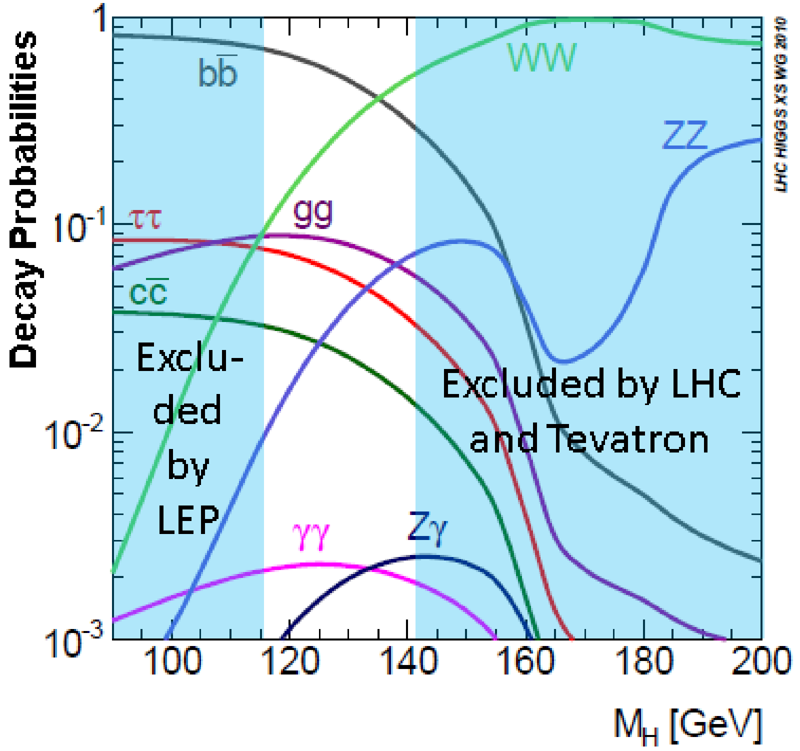




Higgs detection

Branching ratios of higgs decay depend strongly on its mass

- bb is the dominant one, but as it is hadronic has more background
- Gg has similar issue
- WW, where at least one of the W is off-shell, has the problem of being unable to reconstruct the original particle mass. Evidence for a signal can be found by finding excess of events over a background.
- $\tau\tau$ has the problem that τ decays always include at least one neutrino, and therefore it's not possible to reconstruct the invariant mass so well.
- ZZ where at least one Z is offshell, and where both Z decay leptonically to either electrons or muons is called the golden channel. Te cross section is small, but is a very clean channel
- $\gamma\gamma$ is also avery clean channel, even though the signal is even more suppressed than for ZZ



Search channel	m_{max} (GeV)	σ (observed)	μ (m_H = 126 GeV) [Collapse]
$H \to ZZ^* \to 4\ell$	125.0	3.6	1.2 ± 0.6
$H \to \gamma\gamma$	126.5	4.5	1.8 ± 0.5
$H \to WW^* \to \ell\nu\ell\nu$	125.0	2.8	1.3 ± 0.5
Combined	126.5	6.0	1.4 ± 0.3