The Standard Model of Particle Physics

Lecture 4

Global Symmetries and mass terms in QED

We can rewrite the QED lagrangian that includes e, μ, τ, as

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i(\bar{e}, \bar{\mu}, \bar{\tau})D_\mu\gamma^\mu \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} - (\bar{e}, \bar{\mu}, \bar{\tau}) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

If we neglect the last term (the mass term), the lagrangian is invariant under a global $U(3)$ symmetry group transformation:

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \to U \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

Where U is an $U(3)$ matrix. Once one includes the mass term, this symmetry group is broken to $U(1)^3$:

$$e \to e^{i\alpha_e}e, \mu \to e^{i\alpha_\mu}\mu, \tau \to e^{i\alpha_\tau}\tau$$

This is called family lepton number conservation, and we will see that is also present in the Standard Model.

Symmetries of SM Lagrangian (so far)

Without the Yukawa terms, the SM Lagrangian is invariant under the following symmetry group:

$$U(3)^5$$

This is because we can rotate each fermion field (of 3 families) independently.

The SM Lagrangian without the yukawa terms is also CP invariant.

All these symmetries will be (partially) broken b the yukawa couplings.

Yukawa terms: lepton sector

$$\mathcal{L} = \bar{L}_L i\not{D} L_L + \bar{e}_R i\not{D} e_R \tag{4.156}$$

The fields L_L and e_R are not just an $SU(2)_L$ doublet and singlet respectively, they are also in a 3 representation of 2 a (approximate) global symmetry groups, $U(3)_L$, $U(3)_e$. As the operator $\not{D}$ is the identity in flavour space, it is trivial to see that this part of the lagrangian is invariant under such symmetry group.

If we want to add mass terms for the lepton, however, we will end up breaking such symmetry. Now, the mass terms must still come from gauge invariant terms, and need to be the result of symmetry breaking. We need to build an $SU(2)_L \times U(1)_Y$ invariant term, and it needs to include the field L_L and the field e_R. To make it $SU(2)_L$ invariant, we need another $SU(2)_L$ doublet to contract the L_L doublet. This could be the higgs doublet Φ. However, we also need such term to be a singlet under $U(1)_Y$. This would happen only if the higgs doublet would have a specific values of Y, equal to $Y_L - E_e$. Luckily, the value of Y_Φ takes exactly this value! So we can add to the lagrangian a term

$$-y^L_{ij} \bar{L}_{L,i} e_{R,j} \Phi + h.c. \tag{4.157}$$

Yukawa terms: lepton sector

Unlike what we have done so far, where all terms where coming from covariant derivatives, this term is something that we add add hoc, without following the "minimal coupling" principle, but still respecting the gauge symmetry of the lagrangian. In the unitary gauge, where the goldstone bosons decouple, this will take the form

$$-y^L_{ij}\bar{e}_{L,i}e_{R,j}\frac{v+h}{\sqrt{2}} + h.c. \tag{4.158}$$

giving a mass matrix

$$M^e_{ij} = y^L_{ij}\frac{v}{\sqrt{2}} \tag{4.159}$$

The matrix y^L_{ij} is not necessarily an hermitian matrix, and is complex in general. It multiplies different fields on the 2 sides. To diagonalise this matrix, we need to rotate separately the left and right fields. We start by noting that

$$y^L_{ij}(y^L_{jl})^\dagger \tag{4.160}$$

is hermitian, and can be diagonalised by a transformation $SU(3)_{L_L}$:

$$y^L_{ij}(y^L_{jl})^\dagger = U_e D^2_e U^\dagger_e \tag{4.161}$$

Yukawa terms: lepton sector

Similarly,

$$(y^L_{ij})^\dagger y^L_{jl} \tag{4.162}$$

is also hermitian, and can be diagonalised by a transformation $SU(3)_{e_R}$:

$$(y^L_{ij})^\dagger y^L_{jl} = W_e D_e^2 W_e^\dagger \tag{4.163}$$

Note that the diagonal matrix D_e is the same in both cases, as the eigenvalues of the 2 products are the same on both cases. If D_e has only positive eigenvalues, it can be shown that these decompositions are unique, and that

$$y^L = U_e D_e W_e^\dagger \tag{4.164}$$

The matrices $U_e, W_e^\dagger$ will get reabsorbed by the left and right flavour transformations, leaving the diagonal mass matrix

$$M^e = D_e \frac{v}{\sqrt{2}} \tag{4.165}$$

Yukawa terms: lepton sector

All other terms in the lagrangian are invariant separately under both transformations, so no trace of such transformation remains after it is done. So the parameters in U_e, W_e are not observable, and the yukawa lepton sector brings us 3 additional parameters only, the masses m_e, m_μ, m_τ.

The original symmetry group $U(3)_L \times U(3)_R$ is broken. If the masses were all degenerate, the group would break to $U(3)_L \times U(3)_R \to U(3)_V = U(1)_L \times SU(3)_V$ as we have seen previously. Given that the masses are not degenerate, $SU(3)_V$ breaks to the diagonal subgroup

$$U(3)_L \times U(3)_R \to U(1)_e \times U(1)_\mu \times U(1)_\tau \tag{4.166}$$

The symmetry groups $U(1)_l, l = e, \mu, \tau$ are called lepton family number conservation, that are exact in the SM. The fact that the lagrangian has an exact global symmetry under these groups implies that the number of e, μ, τ leptons, N_e, N_μ, N_τ, is separately conserved in each interaction. A subgroup of such group is $U(1)_L$, called lepton number conservation, that implies the conservation of the total lepton number $N_L = N_e + N_\mu + N_\tau$.

Yukawa terms: quark sector

We can try to apply the same method to give mass to quarks. this time, the difference will be that both up and down quarks need to get a mass. for down quarks, once again the value of $Y_Q - Y_d = -\frac{1}{2}$ is the same as before, and we can still couple it to the higgs doublet

$$-y^d_{ij}\bar{Q}_{L,i}d_{R,j}\Phi + h.c. \tag{4.167}$$

In the case of up quarks, however, $Y_Q - Y_u = \frac{1}{2}$. We might think that we are in trouble, but luckily we can use the hermitian conjugate. Note that, to get a gauge invariant term, now the $SU(2)_L$ contraction needs to be different from usual:

$$-y^u_{ij}\bar{Q}_{L,i,a}u_{R,j}\epsilon^{ab}\Phi^\dagger_b + h.c. \tag{4.168}$$

where we made explicit the $SU(2)_L$ indices a, b.

We can operate in the same way as before, obtaining the decompositions

$$y^u = U_u D_u W_u^\dagger \tag{4.169}$$

$$y^d = U_d D_d W_d^\dagger \tag{4.170}$$

Yukawa terms: quark sector

There is only one problem. In the case of quarks, the symmetry group is

$$U(3)_Q \times U(3)_u \times U(3)_d \tag{4.171}$$

So, while we are free to operate separately different rotations W_u, W_d on the right handed fields u_R, d_R, and keeping the rest of the lagrangian invariant, operating different transformations U_u, U_d on the 2 components of the left handed fields will affect some terms of the other parts of the lagrangian. We need to work in the mass eigenstats basis, so we will proceed anyway. We get the mass terms as with leptons:

$$M^u = D_u \frac{v}{\sqrt{2}} \tag{4.172}$$

$$M^d = D_d \frac{v}{\sqrt{2}} \tag{4.173}$$

Yukawa terms: quark sector

By applying the U_u, U_d transformation on left handed fields, we will get a factor

$$V_{CKM} = U_u^\dagger U_d \tag{4.174}$$

in any term of the lagrangian that was contracting a left handed u-family quark with a left handed d-family quark. There is only one such term in the lagranian, the one giving the interaction with the W boson. This allows the weak interaction to change a quark of a family in a quark of any other family. Note that while the W interaction vertex now turns into

$$-i\frac{g}{\sqrt{2}}\gamma^\mu P_L \delta^{ij} \to -i\frac{g}{\sqrt{2}}\gamma^\mu P_L V^{ij} \tag{4.175}$$

thus connecting all up quarks with all down quarks, the interaction vertex with both the photon and the Z boson remains flavour-diagonal, as they connect only up quark with up quarks, and down quarks with down quarks. At tree level, flavour is conserved in SM neutral currents. At loop level, we will see that, thanks to the GIM mechanism.

Yukawa terms: quark sector

Of the original symmetry group, only the subgroup $U(1)_B$ of baryon number conservation survives. One can check, by counting d.o.f., that for n_F families, the CKM matrix has

$$\frac{n_F(N_F-1)}{2} \tag{4.176}$$

real parameters (angles), and

$$\frac{(n_F-1)(n_F-2)}{2} \tag{4.177}$$

phases. For $n_F = 3$ we obtain 3 angles and 1 phase. The existence of a non-zero phase is a source of CP violation in the standard model.

The CKM matrix elements are approximately

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{4.178}$$

Surviving global symmetries

- The surviving global symmetries are just the Baryon number B, lepton number L and lepton family numbers L_i.
- Note that both B and L are anomalous symmetries, while $B-L$ is anomaly free

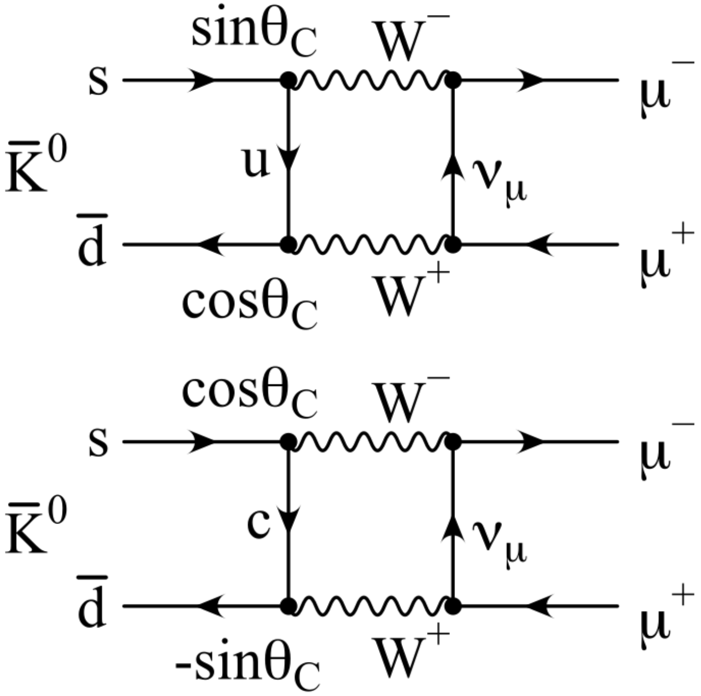
GIM mechanism

- At tree level, neutral currents in the SM have diagonal couplings, $e\bar{e}, \mu\bar{\mu}, u\bar{u}, \ldots$
- At loop level, by exchange of 2 W one gets corrections to neutral currents
- Unitarity of the V_{CKM} matrix suppresses the resulting processes
- Example for K_L^0 $(s\bar{d})$, the diagram is a function of the quark mass in the internal line $f(m_{u,c})$. The c, b, t were not know at the time of formulation of GIM mechanism. The GIM mechanism and the postulation of the c quark were made to explain the rareness of this decay mode.
- So let's consider just a $2x2$ V_{CKM} matrix, the Cabibbo Matrix. The total amplitude is

$$\mathcal{M} = Sin\theta_c Cos\theta_c (f(m_u) - f(m_c))$$

- The function f is a function that actually depends on m_q^2, and that can be expanded in powers of $\frac{m_q^2}{m_W^2}$, when this parameter is small. As a result, the zero order value of the diagram cancels out, and the result is suppressed by a factor

$$Sin\theta_c Cos\theta_c \frac{m_c^2 - m_u^2}{m_W^2}$$



Charm quark prediction

Rareness of $\bar{K}^0 \to \mu^+\mu^-$ explained by formulation of GIM mechanism and postulation of c quark.

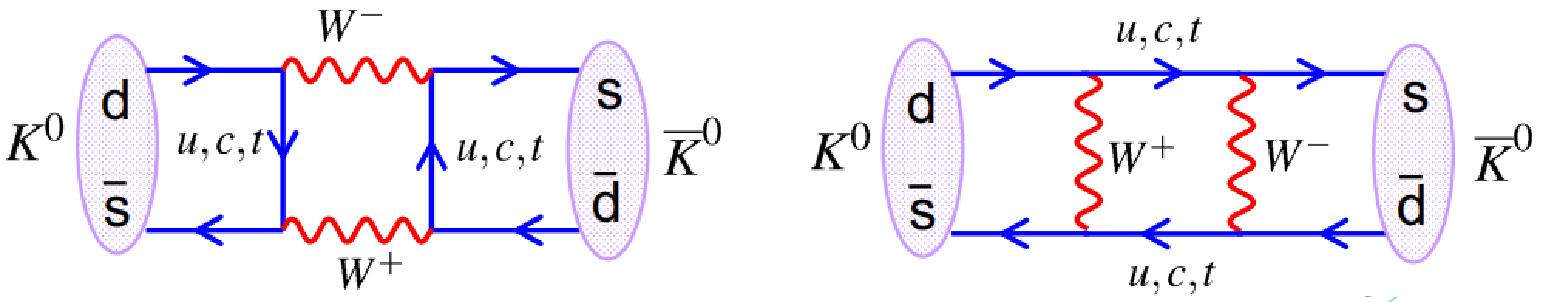
The neutral kaon mixing $K^0 - \bar{K}^0$ was observed. It appeared that $K^0, \bar{K}^0$ were not mass eigenstates, i.e. single propagating particles, but rather admixtures of 2 different particles with very similar mass but very different lifetime:

$$K_S = K_1 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0), K_L = K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$

$$M(K_S) \approx M(K_L) \approx 498MeV, \tau(K_S) \sim 0.9 \times 10^{-10}s, \tau(K_L) \sim 0.5 \times 10^{-7}s$$

The mixture could be of the right size only if $m_c \sim 1.5GeV$, as the GIM mechanism suppression (ignoring the unknown third generation) was

$$sin^2\theta_c cos^2\theta_c \frac{m_c^2 - m_u^2}{m_W^2}$$



CP violation discovery to postulate the third family

K_1 and K_2 had different CP parity. Both particles have negative parity (pseudoscalar mesons). The eigenvalue of C was different however.

$$C(K_1) = \mathrm{C}\left(\frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)\right) = \frac{1}{\sqrt{2}}(\bar{K}^0 - K^0) = -\left(\frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)\right)$$
$$C(K_2) = \mathrm{C}\left(\frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)\right) = \frac{1}{\sqrt{2}}(\bar{K}^0 + K^0) = +\left(\frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)\right)$$

From the conservation of angular momentum, the decay of a K particle to 2 pions led to a 2 pion $CP = +$ state, and so it is allowed only for K_1

The decay of a K particle to 3 pions led to a 3 pion $CP = -$ state, and so it is allowed only for K_2

$$m_K - 2m_\pi = 220MeV, m_K - 3m_\pi = 80MeV$$

More energy available was increasing the “phase space” for the 2 pion decay, thus helping to explain why $K_S = K_1$ had a shorter lifetime

CP violation discovery to postulate the third family

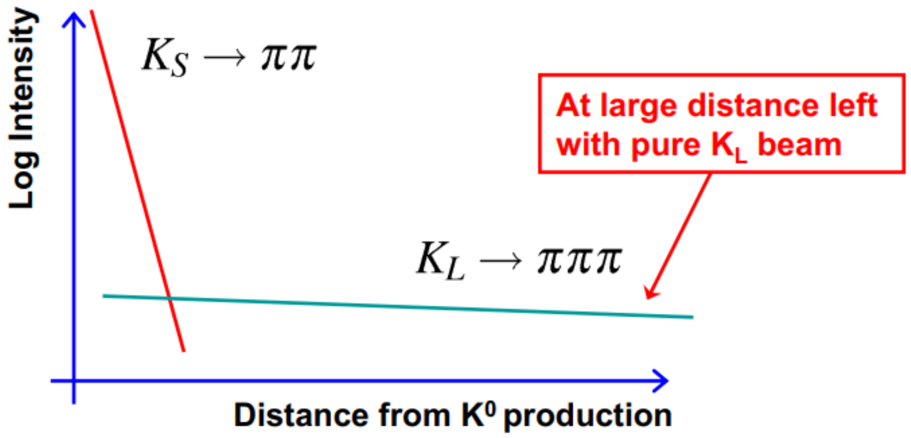
Away from the K^0 production point, one expects to have only K_L

The discovery of $K_L \to \pi\pi$ (1964) led to the conclusion of CP violation in the SM

This happens because the mass eigenstate K_L and the CP eigenstate K_2 are not exactly the same, but rather

$$K_S = \frac{1}{\sqrt{1+\varepsilon^2}}(K_1 + \varepsilon K_2)$$

$$K_L = \frac{1}{\sqrt{1+\varepsilon^2}}(K_2 - \varepsilon K_1)$$



CP violation discovery to postulate the third family

CP violation in the SM could not happen with just 2 families. CP violation requires the CKM matrix to have an imaginary part, a phase, and we saw that the number of phases of a CKM matrix for n_F families is

$$\frac{(n_F - 1)(n_F - 2)}{2}$$

Thus, a third family was required, in order to explain CP violation. This led to the postulation of the third family(1973).

The first 2 particles of the third family were discovered in 1975 (τ) and 1977 (b).

The discovery of $B^0 - \bar{B}^0$ oscillations (1987) led to the hint that the top quark was very massive (if its mass would have been smaller, the GIM mechanism would have made such oscillations too weak to be detected at that time.

FCNC suppression in the SM up sector

In general, in the SM, FCNC in the up sector are very suppressed, because the smallness of the down quark masses leads to a very efficient cancellation of the amplitudes via the GIM mechanism:

$$CKM\ factor\ \times \frac{1}{16\pi^2}\frac{m_b^2 - m_s^2}{m_W^2}$$

The down quark sector is instead the one where the largest FCNC are expected. Their size is still small, but not as small as those in the up sector, as the suppression factor is in this case

$$CKM\ factor\ \times \frac{1}{16\pi^2}\frac{m_t^2 - m_c^2}{m_W^2}$$

Flavor signatures for BSM physics that are often looked for are $b \to s\mu\mu, b \to s\gamma$.