The Standard Model of Particle Physics

Lecture 3

Higgs Field

- In the Standard Model. EW Symmetry breaking is accomplished by a single scalar field. This is the minimal requirement to obtain spontaneous symmetry breaking.
- It is possible to accomplish EW symmetry breaking using scalar sectors that contain more scalars, for example, the 2 higgs doublet model.
- In the SM, the scalar fields is a singlet of color, a doublet of $SU(2)_L$, and it has hypercharge $+1/2$

Higgs Potential

In order to develop a nonzero vev, we need to write down the potential for the scalar field and make it such that the minimum is located at a nonzero value of the field. Due to gauge invariance, the only gauge invariant combination that can appear in the lagrangian is

$$\Phi\Phi^\dagger \tag{4.132}$$

So the most general potential will be

$$V(\Phi) = \mu^2\Phi\Phi^\dagger + \lambda(\Phi\Phi^\dagger)^2 \tag{4.133}$$

If $\mu^2 > 0$ the potential has a minimum for $\Phi = 0$ and develops no vev: the theory remains unbroken. So we need to change the sign of μ^2. As we like to keep μ^2 positive, let's add a $-$ sign in front

$$V(\Phi) = -\mu^2\Phi\Phi^\dagger + \lambda(\Phi\Phi^\dagger)^2 \tag{4.134}$$

Now the potential develops a minimum for a nonzero value of Φ. Derivating w.r.t Φ we get

$$\frac{\partial V}{\partial \Phi} = \Phi^\dagger \left(-\mu^2 + 2\lambda\Phi\Phi^\dagger\right) \tag{4.135}$$

Higgs Potential

The minimum therefore has to satisfy

$$\frac{\partial V}{\partial \Phi}(\Phi_0) = \Phi_0^\dagger\left(-\mu^2 + 2\lambda\Phi_0\Phi_0^\dagger\right) = 0 \tag{4.136}$$

$$-\mu^2 + 2\lambda\Phi_0\Phi_0^\dagger = 0 \tag{4.137}$$

$$\Phi_0\Phi_0^\dagger = \frac{\mu^2}{2\lambda} \tag{4.138}$$

The combination $\Phi_0\Phi_0^\dagger$ is invariant under gauge transformations. However, we have taken a well defined value in the previous section for Φ_0:

$$\Phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \tag{4.139}$$

so we get

$$v^2 = \frac{\mu^2}{\lambda} \tag{4.140}$$

We now want to find the higgs mass. We may also want to check the masses of the Goldstone particles $G^\pm, G^0$. For this, we can, as usual, get the mass matrix $(a, b = (h, G^0, G^+))$

$$M_{ab} = \frac{\partial^2 V}{\partial a \partial b^\dagger}|_{\Phi=\Phi_0} = \begin{pmatrix} 2\lambda v^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4.141}$$

So we indeed confirm that all goldstone bosons are massless, and that

$$m_h^2 = 2\lambda v^2 \tag{4.142}$$

How to derive the mass matrix

- Rewrite the field as $\Phi = \begin{pmatrix} G^+ \\ \frac{v+h+iG^0}{\sqrt{2}} \end{pmatrix}$
- Rewrite the potential $V = -\mu^2\Phi\Phi^\dagger + \lambda(\Phi\Phi^\dagger)^2$ in terms of G^+, G^-, h, G^0 fields
- Take the second derivatives

$$\frac{\partial^2 V}{\partial a \partial b^\dagger}, a, b = \text{of } G^+, G^-, h, G^0$$

- Get the value of the matrix when the vev of the fields G^+, G^-, h, G^0 is zero

Activity Break

Find the mass matrix and check that it contains the right number of zero eigenvalues (what does Goldstone Theorem tell us?) and the right value of the higgs mass

Kinetic Higgs term and EW Symmetry Breaking

To find the masses of the EW gauge bosons in terms of v, all we need to do is to write the kinetic term for the new field and isolate the mass terms coming from the covariant derivative:

$$\begin{aligned}
D_\mu &= \partial_\mu - igW^a_\mu t^a - ig' Y_\Phi B_\mu && (4.103)\\
&= \partial_\mu - igW^a_\mu t^a - ig' \frac{1}{2} B_\mu && (4.104)\\
&= \begin{pmatrix} \partial_\mu - i\frac{1}{2}gW^3_\mu - i\frac{1}{2}g' B_\mu & -i\frac{1}{2}g(W^1_\mu - iW^2_\mu) \\ -i\frac{1}{2}g(W^1_\mu + iW^2_\mu) & \partial_\mu + i\frac{1}{2}gW^3_\mu - i\frac{1}{2}g' B_\mu \end{pmatrix} && (4.105)\\
\mathcal{L}_{kin} &= D_\mu \Phi D^\mu \Phi^\dagger && (4.106)
\end{aligned}$$

We can group the $W^{1,2}$ real fields into complex fields with ± 1 electric charge as

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu) \qquad (4.107)$$

We get

$$\begin{aligned}
D_\mu &= \begin{pmatrix} \partial_\mu - i\frac{1}{2}gW^3_\mu - i\frac{1}{2}g' B_\mu & -i\frac{1}{\sqrt{2}}gW^+_\mu \\ -i\frac{1}{\sqrt{2}}gW^-_\mu & \partial_\mu + i\frac{1}{2}gW^3_\mu - i\frac{1}{2}g' B_\mu \end{pmatrix} && (4.108)\\
\Phi &= \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix} && (4.109)
\end{aligned}$$

Kinetic Higgs term and EW Symmetry Breaking

Now, the mass terms will need to include 2 powers of the gauge fields W, B, and a coefficient of dimensions of squared energy, so they cannot contain derivatives. So we can drop the derivative part. We can also drop all fields inside Φ, as they will not appear in the mass terms for the gauge bosons, so

$$\mathcal{L}_{W,Zmass} = \left| \begin{pmatrix} -i\frac{1}{2}gW^3_\mu - i\frac{1}{2}g'B_\mu & -i\frac{1}{\sqrt{2}}gW^+_\mu \\ -i\frac{1}{\sqrt{2}}gW^-_\mu & +i\frac{1}{2}gW^3_\mu - i\frac{1}{2}g'B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix} \right|^2 \tag{4.110}$$

$$= \begin{pmatrix} -i\frac{gv}{2}W^+_\mu \\ -i\frac{v}{2\sqrt{2}}\left(g'B_\mu - gW^3_\mu\right) \end{pmatrix} \begin{pmatrix} i\frac{gv}{2}W^-_\mu \\ i\frac{v}{2\sqrt{2}}\left(g'B_\mu - gW^3_\mu\right) \end{pmatrix} \tag{4.111}$$

$$= \begin{pmatrix} -i\frac{gv}{2}W^+_\mu \\ -i\frac{v\sqrt{g^2+(g')^2}}{2\sqrt{2}}\frac{g'B_\mu - gW^3_\mu}{\sqrt{g^2+(g')^2}} \end{pmatrix} \begin{pmatrix} i\frac{gv}{2}W^-_\mu \\ i\frac{v\sqrt{g^2+(g')^2}}{2\sqrt{2}}\frac{g'B_\mu - gW^3_\mu}{\sqrt{g^2+(g')^2}} \end{pmatrix} \tag{4.112}$$

We can identify already the properly normalised linear combination of W^3, B that will make up the massive Z boson as:

$$Z_\mu = \frac{g'B_\mu - gW^3_\mu}{\sqrt{g^2+(g')^2}} \tag{4.113}$$

The orthogonal component, the photon, will instead remain massless

$$A_\mu = \frac{gB_\mu + g'W^3_\mu}{\sqrt{g^2+(g')^2}} \tag{4.114}$$

Kinetic Higgs term and EW Symmetry Breaking

When one has multiple fields, it might not be as easy to identify the mass eigenstates from the expression. One therefore writes the "mass matrix", for the fields a, b, as

$$M_{ab} = \frac{\partial^2 \mathcal{L}}{\partial a \partial b^\dagger} \tag{4.115}$$

Such mass matrix is always block diagonal, with one block for each possible value of the electric charge. Neutral scalars, if the CP symmetry is conserved, are also separated into 2 different blocks, one for CP even ($CP = +$) and one for CP odd ($CP = -$). We can indeed check that, for $a = W^+, W^3, B$ our mass matrix is

$$\begin{pmatrix} \frac{g^2v^2}{4} & 0 & 0 \\ 0 & \frac{g^2v^2}{4} & -\frac{gg'v^2}{4} \\ 0 & -\frac{gg'v^2}{4} & \frac{(g')^2v^2}{4} \end{pmatrix} \tag{4.116}$$

The first column and row refer to a particle of charge $Q + \pm 1$, and is indeed block diagonal. The remaining 2×2 block can be diagonalised, with the eigenvalues giving the values of the masses, and the relative (normalised) eigenvectors giving the right linear combinations that generate the mass eigenstates. Note that the definition of the mass matrix automatically account for the $1/2$ factor in the case of real fields. The diagonalised matrix becomes

$$\begin{pmatrix} \frac{g^2v^2}{4} & 0 & 0 \\ 0 & \frac{(g^2+(g')^2)v^2}{4} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4.117}$$

Kinetic Higgs term and EW Symmetry Breaking

From the mass matrix we can immediately read the mass values:

$$M_W = \frac{g}{2}v \tag{4.118}$$

$$M_Z = \frac{\sqrt{g^2+(g^{'})^2}}{2}v \tag{4.119}$$

$$M_A = 0 \tag{4.120}$$

Substituting back into the lagragian the expressions for B, W^3, one can obtain the couplings of the fermions to the Z, A gauge bosons. Moreover, the other terms that we have discarded coming from the kinetic term of the Higgs field will also generate interactions between the gauge fields and the higghs field h. We will also get interaction between gauge fields, higgs field and the goldstone bosons $G^{\pm}, G^0$. We will need to understand what those term mean and if they are physical. We can rewrite the covariant derivative for a generic field charged under $SU(2)_L \times U(1)_Y$ in terms of the mass eigenstates as

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+ t^+ W_\mu^- t^-\right) - i\frac{g^2T^3-(g^{'})^2Y}{\sqrt{g^2+(g^{'})^2}}Z_\mu - i\frac{gg^{'}\left(T_3+Y\right)}{\sqrt{g^2+(g^{'})^2}}A_\mu \tag{4.121}$$

$$= \partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+ t^+ W_\mu^- t^-\right) - i\frac{g^2T^3-(g^{'})^2Y}{\sqrt{g^2+(g^{'})^2}}Z_\mu - ieQA_\mu \tag{4.122}$$

where we have identified

$$Q = T^3 + Y \tag{4.123}$$

Kinetic Higgs term and EW Symmetry Breaking

$$e = \frac{gg'}{\sqrt{g^2 + (g')^2}} \tag{4.124}$$

To change the base from the "flavour" base to the mass eigenstate base, it is convenient to define the angle θ_w:

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix} \tag{4.125}$$

with

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + (g')^2}} = \frac{M_W}{M_Z} \tag{4.126}$$

$$\sin\theta_w = \frac{g'}{\sqrt{g^2 + (g')^2}} \tag{4.127}$$

the coupling to Z can be rewritten in terms of Q rather than Y:

$$g^2T^3 - (g')^2Y = (g^2 + (g')^2)T^3 - (g')^2Q \tag{4.128}$$

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ t^+ W_\mu^- t^-) - i\frac{g}{\cos\theta_w}(T^3 - \sin^2\theta_w Q)Z_\mu - ieQA_\mu \tag{4.129}$$

with

$$e = \frac{g}{\sin\theta_w} \tag{4.130}$$

by taking the low energy limit, we can connect g, M_W with the fermi constant

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \tag{4.131}$$

Gauge fixing and Propagators for W,Z and Goldstone bosons

$$\mathcal{L} = -\frac{1}{2}A^a_\mu\left((-g^{\mu\nu}\partial^2 + (1-\frac{1}{\xi})\partial^\mu\partial^\nu)\delta^{ab} - M^{ab}\right)A^b_\nu + \frac{1}{2}G^i\left(-\partial^2 - \xi M^{ij}\right)G^j \qquad (4.149)$$

this gives us the propagators of the theory. For the scalars, the gauge fixing term gives them a mass

$$M^2_{G^0} = \xi M^2_Z \qquad (4.150)$$
$$M^2_{G^\pm} = \xi M^2_W \qquad (4.151)$$

As the mass is gauge-dependent, it is clear that such particles must be unphysical. However, for a generic gauge ξ we must include the feynman diagrams that include such unphysical particles as internal lines, as much as we do for ghosts, to retain gauge invariance. One special exception is, at tree level, the limit $\xi \to \infty$. In this limit, the particles become infinitely massive and decouple from the theory, as there is no vertex that is $\propto \xi$. This is called the unitary gauge. With a bit of work, we can also get the propagator for the W, Z fields

$$\frac{-i}{q^2 - M^2}\left(g^{\mu\nu} - (1-\xi)\frac{q^\mu q^\nu}{q^2 - \xi M^2}\right) \qquad (4.152)$$

In the unitary gauge, this simplifies to

$$\frac{-i}{q^2 - M^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M^2}\right) \qquad (4.153)$$

Similarly, the sums over the external polarizations will now take the form

$$\sum \epsilon^\mu \epsilon^{\nu,*} = -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M^2}\right) \qquad (4.154)$$

Goldstone Equivalence Theorem

- Goldstone Theorem:
- For a spontaneously broken Global symmetry, number of broken generators = number of goldstone bosons.
- For a spontaneously broken Gauge (local) symmetry, number of broken generators = number of gauge bosons that acquire mass
- The number of degrees of freedom is conserved:
- Initial state has n real scalars and m massless vectors (that have $2m$ degrees of freedom), $n + 2m$ degrees of freedom
- Final state has $n - m$ real scalars and m massive vectors (that have $3m$ degrees of freedom), $n - m + 3m = n + 2m$ degrees of freedom

Goldstone Equivalence Theorem

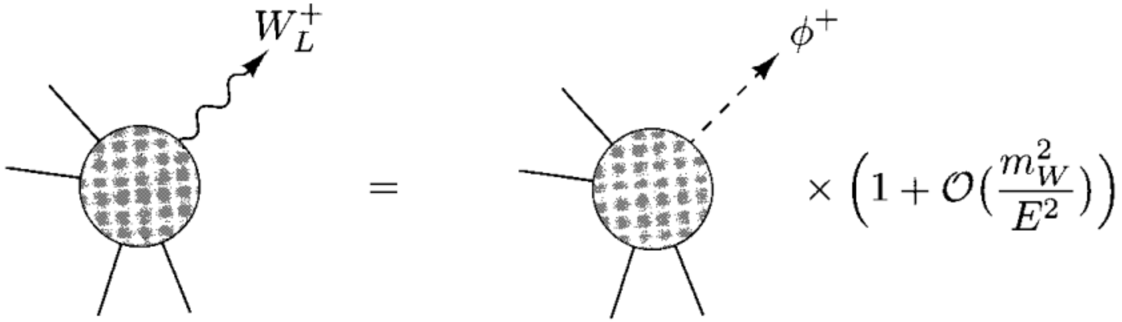


Figure 21.3. The Goldstone boson equivalence theorem. At high energy, the amplitude for emission or absorption of a longitudinally polarized massive gauge boson becomes equal to the amplitude for emission or absorption of the Goldstone boson that was eaten by the gauge boson.

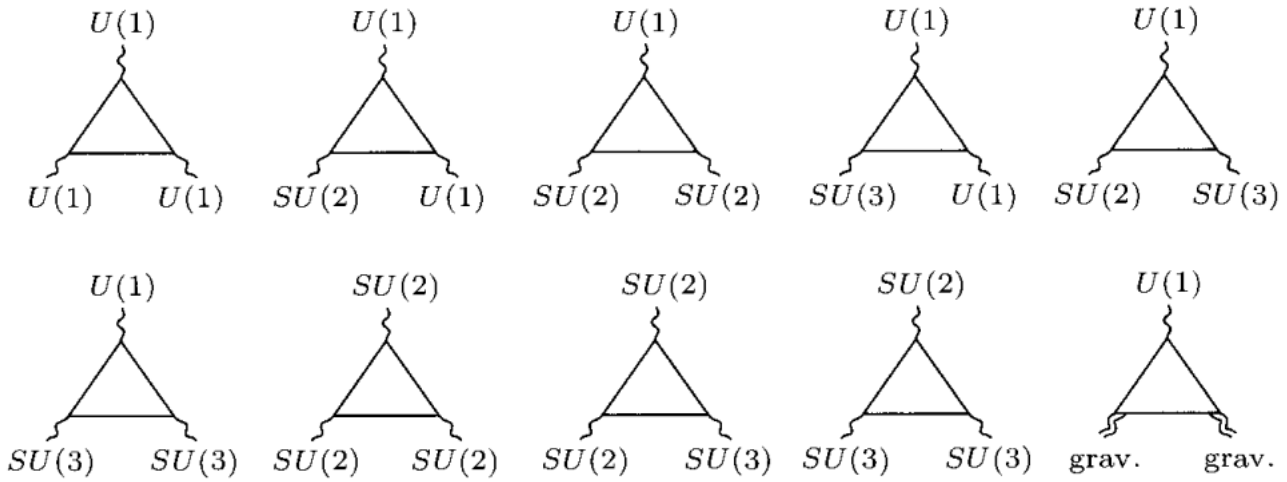
Top quark decay

- Verify the Goldstone Equivalence theorem for the decay of a very heavy top quark
- Feynman rules to use:
- Top-bottom-W interaction $i\frac{g}{\sqrt{2}}\gamma_\mu P_L$
- Top-bottom-G^- interaction $i\frac{g}{\sqrt{2}}\frac{m_t}{m_W}P_R$
- Compare the squared matrix elements and verify they match at first order in $\frac{m_t}{m_W}$

Gauge Anomalies in the SM

- Forconvenience, we can take all fermions to be right handed, by replacing the particles with the relativeantiparticles for any left handed fermion representation. This will flip all $U(1)$ charges:

F	$SU(3)$	$SU(2)$	$U(1)$
$\bar{Q}_L$	3	2	$-1/6$
u_R	3	1	$2/3$
d_R	3	1	$-1/3$
$\bar{L}_L$	1	2	$1/2$
e_R	1	1	-1



Gauge Anomalies in the SM

There are 10 possible combination, however most of them automatically cancel.

1. $SU(3)^3$ cancels as QCD is not chiral
2. $SU(2)^3$ cancels as it is a special property of $SU(2)$
3. Any combination including either a single $SU(3)$ of $SU(2)$ factor will be proportional to the trace of a single generator, that always vanishes.

This leaves only 3 non-trivial factors to check. The first one is $SU(3)^2 \times U(1)$:

$$\frac{1}{2}\left(2(-\frac{1}{6}) + \frac{2}{3} - \frac{1}{3}\right) = 0 \tag{4.96}$$

The second factor is $SU(2)^2 \times U(1)$:

$$\frac{1}{2}\left(3(-\frac{1}{6}) + \frac{1}{2}\right) = 0 \tag{4.97}$$

Finally, the last factor to check out is $U(1)^3$

$$6(-\frac{1}{6})^3 + 3(\frac{2}{3})^3 + 3(-\frac{1}{3})^3 + 2(\frac{1}{2})^3 + (-1)^3 = -\frac{1}{36} + \frac{8}{9} - \frac{1}{9} + \frac{1}{4} - 1 = 0 \tag{4.98}$$

Gravitational anomaly: $Tr[Y] = -2\left(-\frac{1}{2}\right) + (-1) - 3\left(2\left(\frac{1}{6}\right) - \left(\frac{2}{3}\right) - \left(-\frac{1}{3}\right)\right) = 0$