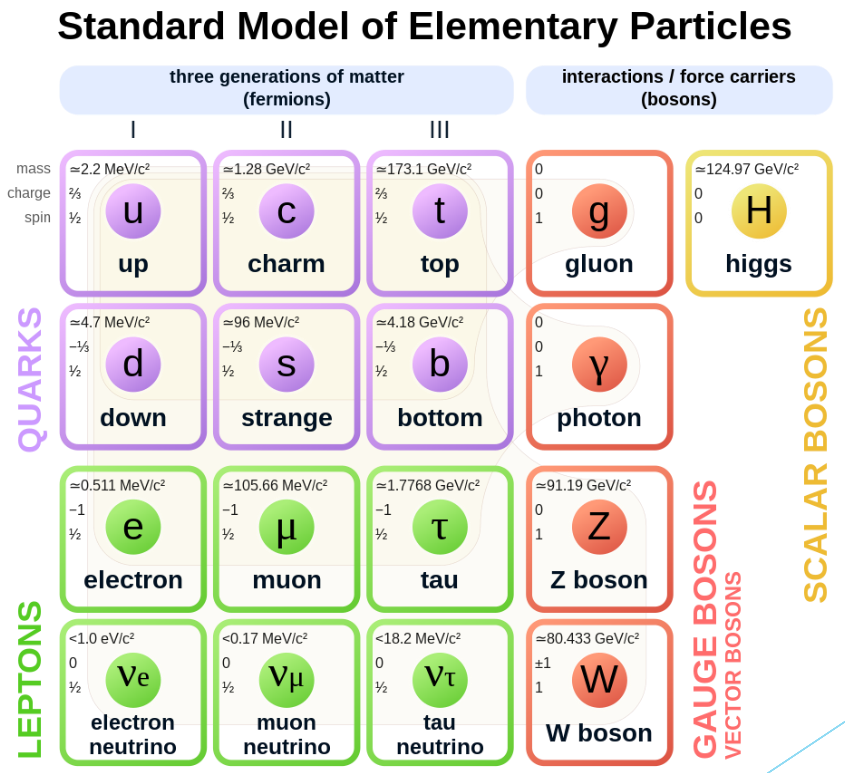
The Standard Model of Particle Physics

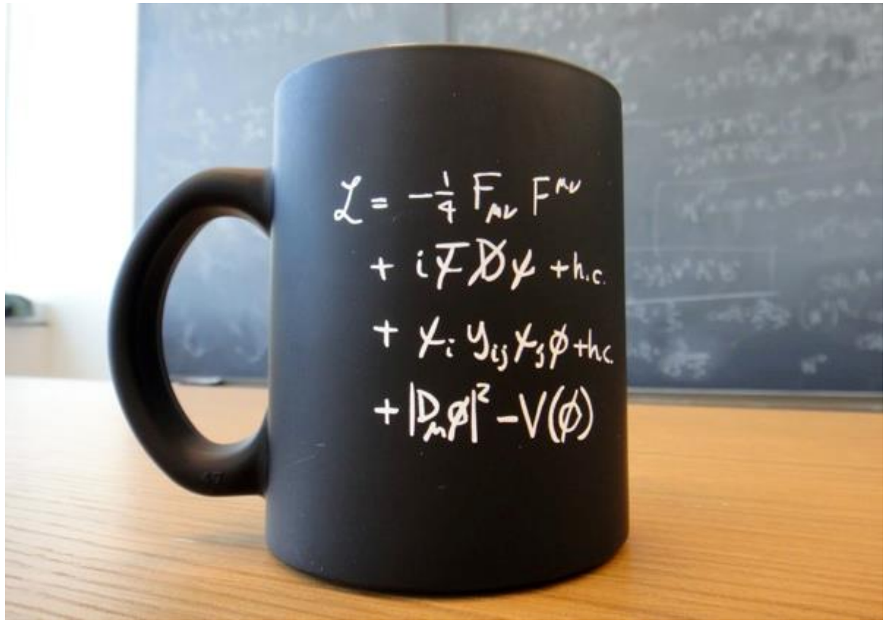
Lecture 1

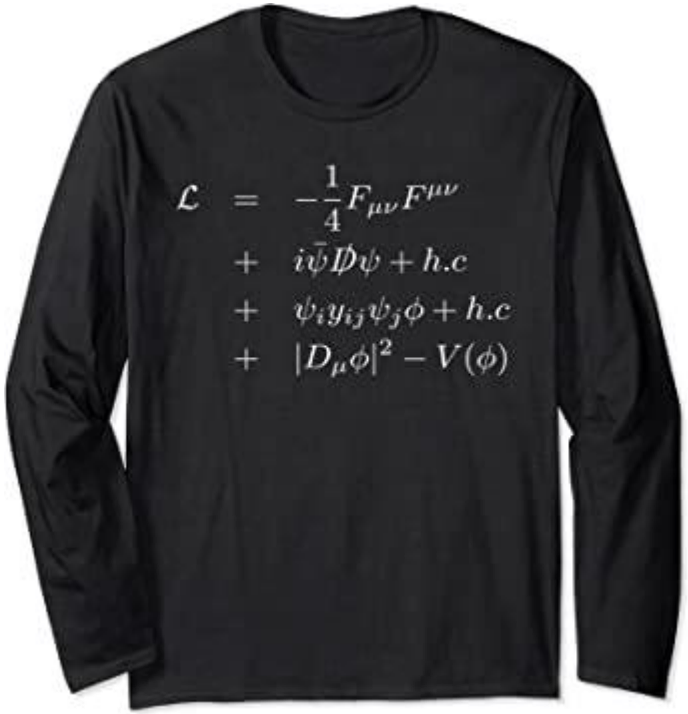
Standard Model Elementary Particles

- Matter: spin ½ fermions
- Force carriers: spin 1 gauge bosons and spin 0 Higgs boson
- Spin 1 bosons related to 3 of the 4 forces of nature: Electromagnetic, Weak, and Strong forces
- Matter (fermions) divided in 2 groups, based on interactions: leptons and quarks
- Leptons do not interact through the Strong force and exist individually
- Quarks interact via the Strong force and do not exist on their own, but rather form Hadrons (Baryons and Mesons)
- Minimal theory to explain Electroweak and Strong forces

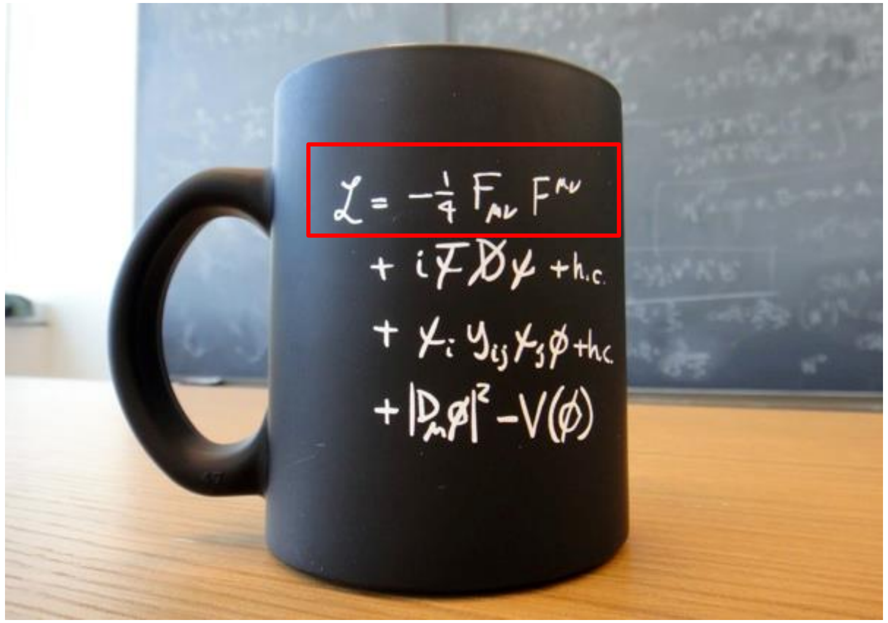


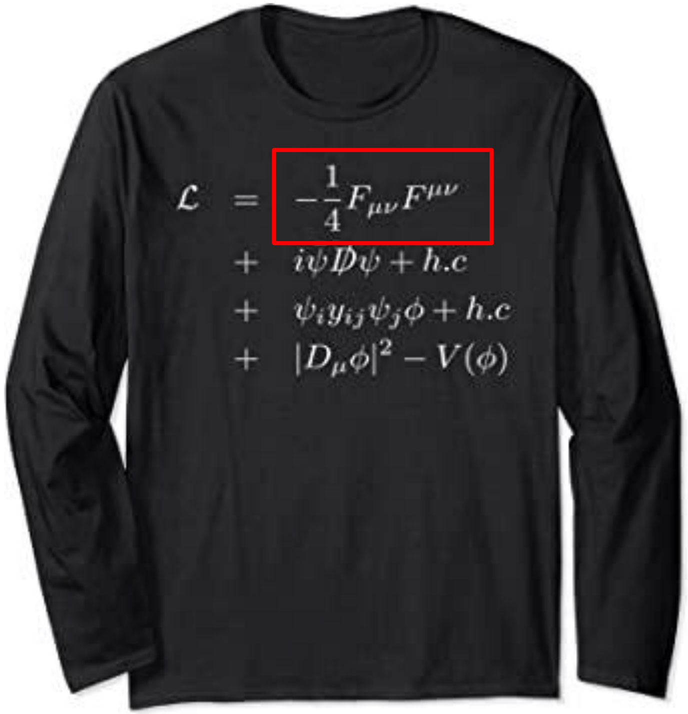
Standard Model Lagrangian





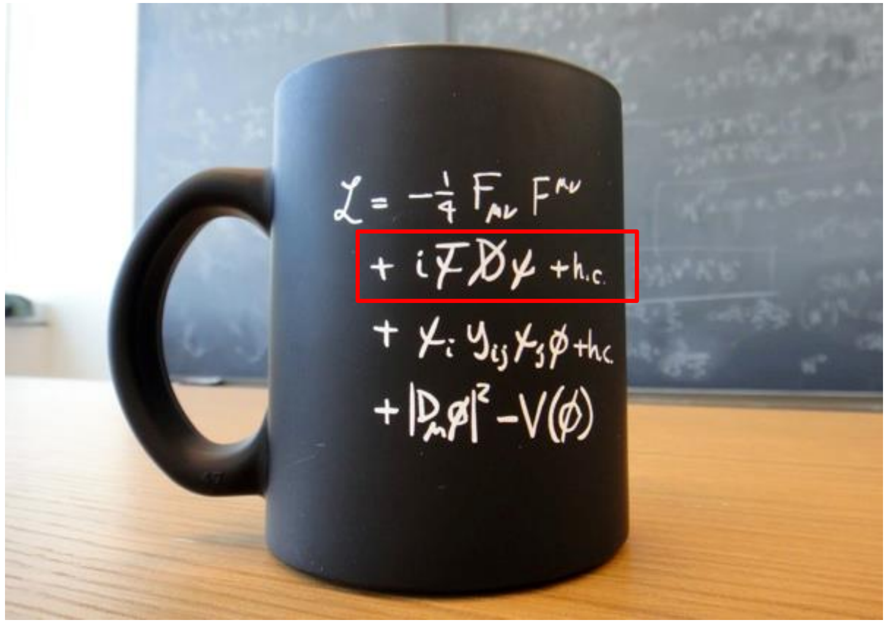
Standard Model Lagrangian

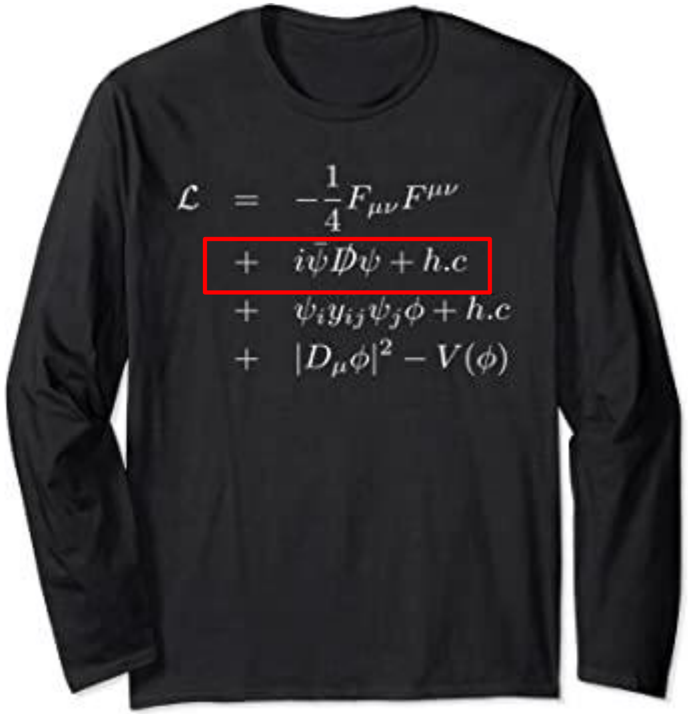




$$\mathcal{L} = \mathcal{L}_{gauge} +$$

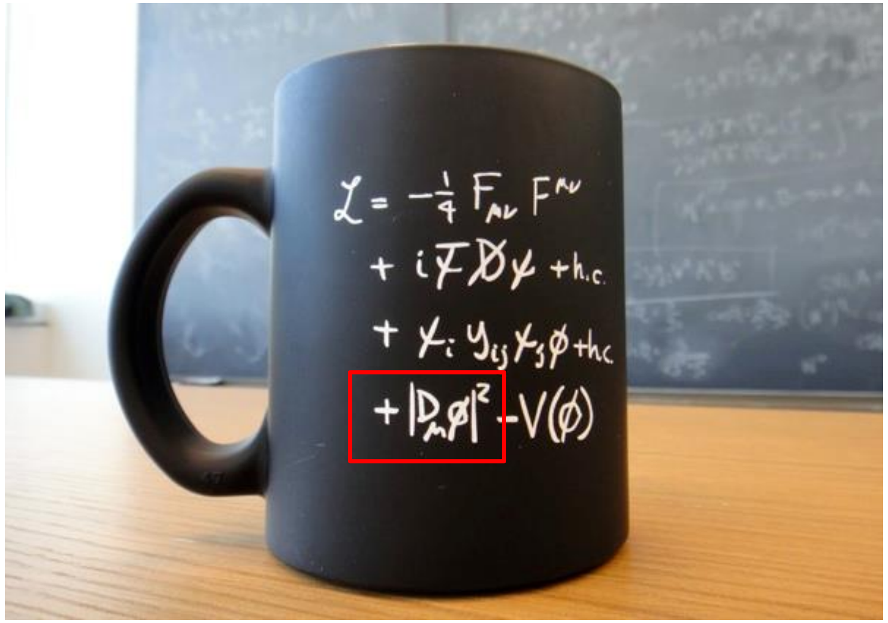
Standard Model Lagrangian





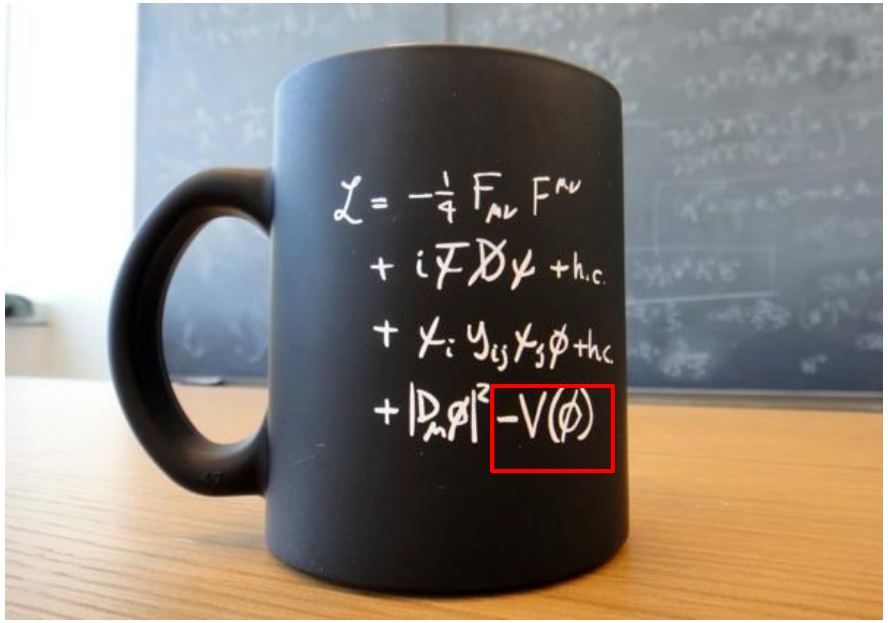
$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} +$$

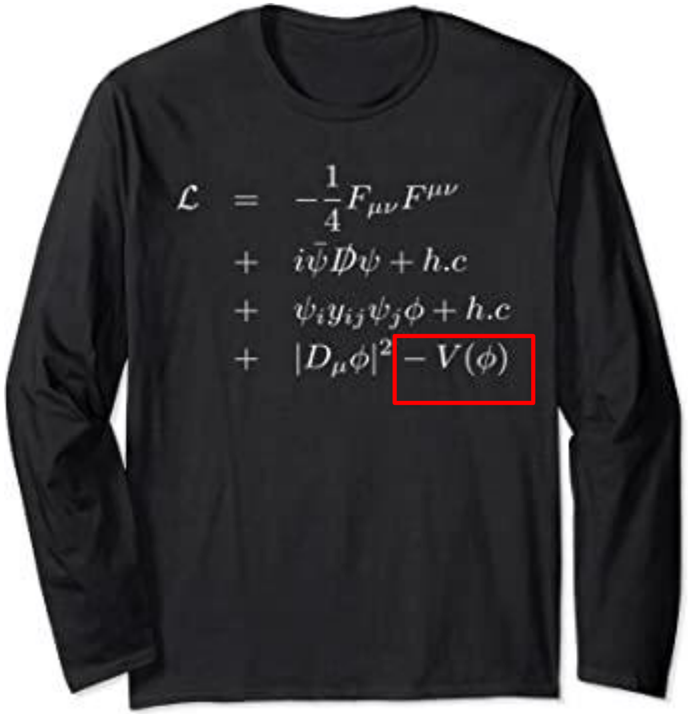
Standard Model Lagrangian



$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{higgs} +$$

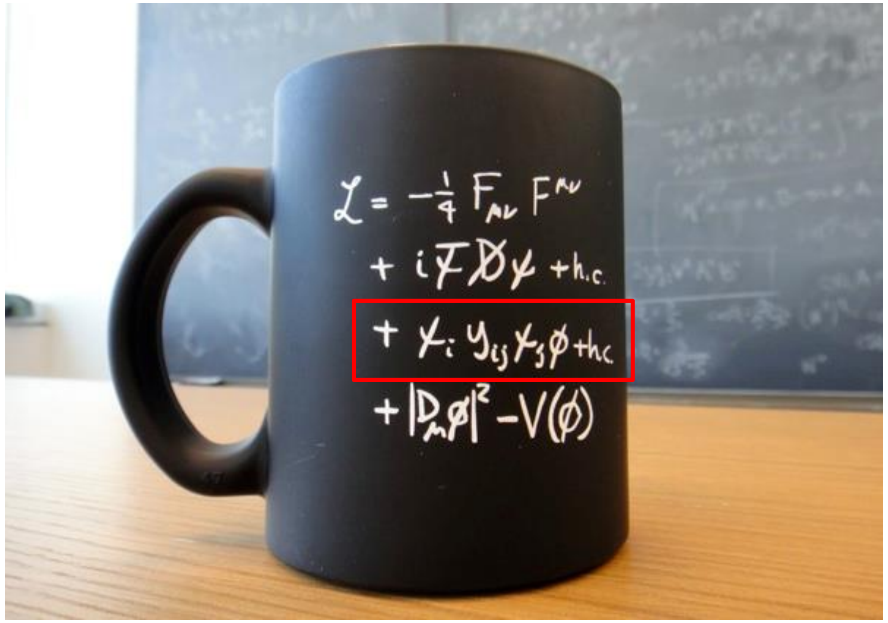
Standard Model Lagrangian

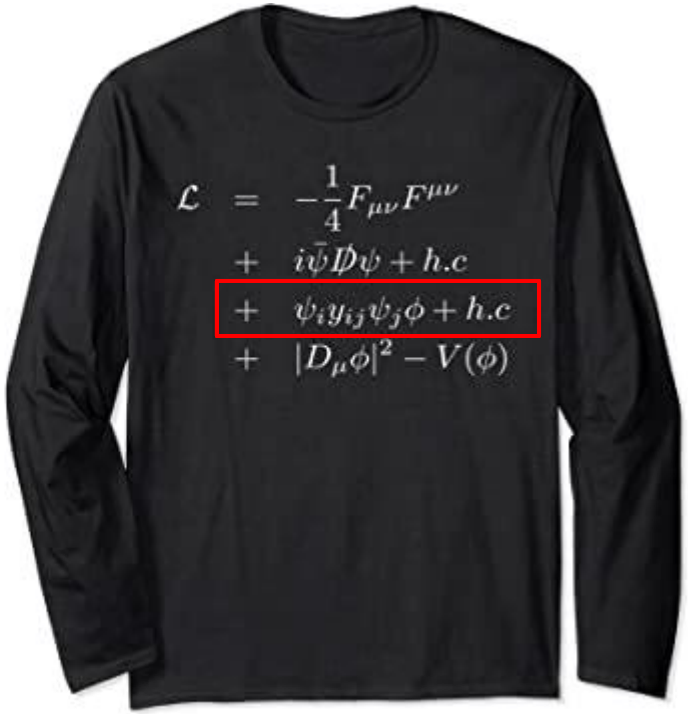




$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{higgs} + \mathcal{L}_{potential} +$$

Standard Model Lagrangian





$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{higgs} + \mathcal{L}_{potential} + \mathcal{L}_{yukawa}$$

Standard Model Gauge Group

- The Gauge Group defines the group under which SM particles will transform under a gauge transformation
- The Gauge Group of the Standard Model is
- $SU(3)_c \times SU(2)_L \times U(1)_Y$
- Field G associated to $SU(3)_c$
- Field W associated to $SU(2)_L$
- Field B associated to $U(1)_Y$
- This Gauge symmetry group will get spontaneously broken to
- $SU(3)_c \times U(1)_Q$
- Gauge group is product of 3 simple groups, gauge term in the lagrangian will be the sum of a gauge term for each simple group

Gauge term for Abelian Group: Covariant Derivative

We consider a fermion field Ψ with electric charge Q. We will say that Ψ belongs to the Q representation of $U(1)_Q$, and it will transform as

$$\Psi \rightarrow e^{ieQ\alpha(x^\mu)}\Psi$$

An arbitrary phase shit that depends on the space-time point is not problematic for $\bar{\Psi}\Psi$ terms, as

$$\bar{\Psi}\Psi \rightarrow \bar{\Psi}\Psi$$

The terms that have a problem are the ones involving a derivative. The derivative along a direction n^μ is

$$n^\mu \partial_\mu \Psi = \lim_{\epsilon \rightarrow 0} \frac{\Psi(x^\mu + \epsilon n^\mu) - \Psi(x^\mu)}{\epsilon}$$

This is a difference of Ψ at DIFFERENT space-time points, that means the difference of fields that transform differently under the phase shift, as the first one will acquire a phase $\alpha(x^\mu + \epsilon n^\mu)$, while the second will acquire a phase $\alpha(x^\mu)$. To obtain an object that transforms correctly, we need to define a transport operator that transforms as

$$U(y,x) \rightarrow e^{ieQ\alpha(y)}U(y,x)e^{-ieQ\alpha(x)}$$

And has the properties

$$U(x,x) = 1, \|U\| = 1$$

where the last equation means that U can (should) be takes as a pure phase. Following what we have seen so far, we can see that a good choice can be

$$U(y,x) = e^{ieQ\int_x^y A_\mu dl^\mu}$$

Gauge term for Abelian Group: Covariant Derivative

Using this operator we get that

$$U(x^\mu + \epsilon n^\mu, x^\mu)\Psi(x^\mu)$$

transforms in the same way as

$$\Psi(x^\mu + \epsilon n^\mu)$$

so we can define

$$\begin{aligned} n^\mu D_\mu \Psi &= \lim_{\epsilon \to 0} \frac{\Psi(x^\mu + \epsilon n^\mu) - U(x^\mu + \epsilon n^\mu, x^\mu)\Psi(x^\mu)}{\epsilon} \\ &= \lim_{\epsilon \to 0} \frac{\Psi(x^\mu + \epsilon n^\mu) - e^{ieQ\epsilon n^\mu A_\mu}\Psi(x^\mu)}{\epsilon} \\ &= \lim_{\epsilon \to 0} \frac{\Psi(x^\mu) + \epsilon n^\mu \partial_\mu \Psi - (1 + ieQ\epsilon n^\mu A_\mu)\Psi(x^\mu)}{\epsilon} \\ &= n^\mu(\partial_\mu - ieQA_\mu)\Psi \end{aligned}$$

So

$$D_\mu = \partial_\mu - ieQA_\mu$$

Gauge term for Abelian Group: Gauge kinetic term

The covariant derivative transforms as

$$D_\mu \rightarrow U D_\mu U^\dagger, U = e^{-ieQ\alpha(x)}$$

And has the property that

$$[D_\mu, D_\nu] = -ieF_{\mu\nu}$$

In the abelian case $F_{\mu\nu}$ is invariant:

$$F_{\mu\nu} \rightarrow U F_{\mu\nu} U^\dagger = U U^\dagger F_{\mu\nu} = F_{\mu\nu}$$

In the abelian case, the gauge kinetic term is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Expanding it in terms of the field A_μ, there are only terms that contain the field 2 times. These terms contribute to the propagator, there are no self-interactions.

Gauge term for Abelian Group: Feynman rules

The propagator is

$$\frac{-ig_{\mu\nu}}{k^2 + i\varepsilon}$$

Gauge term for non-Abelian Group: Covariant Derivative

The relevant difference, comparing to the abelian case, is that now U is not just a phase, but an $SU(N)$ unitary matrix:

$$U(y,x) = e^{ig\int_x^y A_\mu^a t^a dl^\mu}$$

This means that we will need multiple A_μ^a fields.

The covariant derivative is also a matrix:

$$D_\mu = \mathbb{I}\partial_\mu - ieQA_\mu^a t^a$$

Or, by expressing explicitly the matrix indices

$$D_{\mu,bc} = \mathbb{I}_{bc}\partial_\mu - ieQA_\mu^a t_{bc}^a$$

Gauge term for non-Abelian Group: Gauge kinetic term

The main difference comparing to the abelian case is that now $F_{\mu\nu}$ is not invariant:

$$F_{\mu\nu} \rightarrow U F_{\mu\nu} U^{\dagger} \neq F_{\mu\nu}$$

Moreover, $F_{\mu\nu}$ is now a matrix, so we need to adapt out definition for the gauge kinetic term.

The field strength tensor can be rewritten as

$$F_{\mu\nu} = F_{\mu\nu}^{a} t^{a}$$

A contraction of 2 tensors is not invariant, but its trace is:

$$F_{\mu\nu} F^{\mu\nu} \rightarrow U F_{\mu\nu} F^{\mu\nu} U^{\dagger}$$

$$Tr\left[U F_{\mu\nu} F^{\mu\nu} U^{\dagger}\right] = Tr\left[U^{\dagger} U F_{\mu\nu} F^{\mu\nu}\right] = Tr\left[F_{\mu\nu} F^{\mu\nu}\right]$$

Using the expansion, we get

$$Tr\left[F_{\mu\nu} F^{\mu\nu}\right] = F_{\mu\nu}^{a} F_{b}^{\mu\nu} Tr\left[t^{a} t^{b}\right] = \frac{1}{2} F_{\mu\nu}^{a} F_{b}^{\mu\nu} \delta_{ab}$$

Each a component is associated to a different gauge boson, so an $SU(N)$ group will have $N^2 - 1$ gauge bosons, as much as the generators.

$$\mathcal{L} = -\frac{1}{2} Tr\left[F_{\mu\nu} F^{\mu\nu}\right] = -\frac{1}{4} F_{\mu\nu}^{a} F_{a}^{\mu\nu}$$

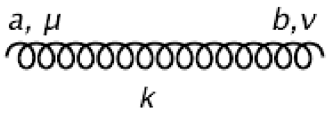
Expanding a single component, we get:

$$F_{\mu\nu}^{a} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} + g f_{abc} A_{\mu}^{b} A_{\nu}^{c}$$

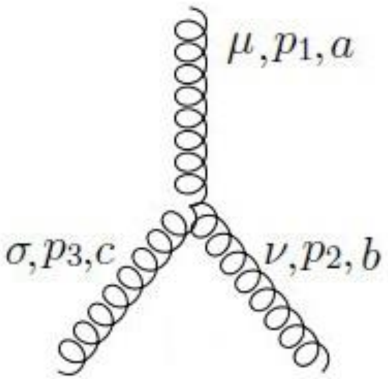
Gauge term for non-Abelian Group: Feynman rules

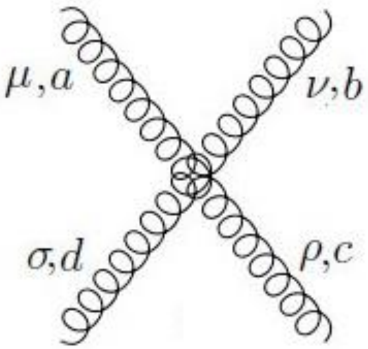
The propagator is

$$\frac{-ig_{\mu\nu}}{k^2 + i\varepsilon}\delta_{ab}$$



However, there are also terms that include 3 and 4 fields in the contraction of $F_{\mu\nu}F^{\mu\nu}$: these terms generate gauge fields self interactions:





Break

Chiral Fermions

Free Fermion Lagrangian is

$$\mathcal{L} = \bar{\Psi} i \partial_\mu \gamma^\mu \Psi - m \bar{\Psi} \Psi$$

We can decompose fermions in left handed and right handed fermions

$$\Psi = \left(\frac{1+\gamma^5}{2}\right)\Psi + \left(\frac{1-\gamma^5}{2}\right)\Psi = \Psi_R + \Psi_L$$

$$\Psi_R = \left(\frac{1+\gamma^5}{2}\right)\Psi = P_R\Psi, \Psi_L = \left(\frac{1-\gamma^5}{2}\right)\Psi = P_L\Psi$$

$P_{R,L}$ are projectors: $P_{R,L}^2 = P_{R,L}, P_R P_L = P_L P_R = 0.$

We can rewrite the lagrangian using $\Psi = \Psi_R + \Psi_L = P_R \Psi_R + P_L \Psi_L$

$$\mathcal{L} = \bar{\Psi}_R i \partial_\mu \gamma^\mu \Psi_R + \bar{\Psi}_L i \partial_\mu \gamma^\mu \Psi_L - m(\bar{\Psi}_R \Psi_L + \bar{\Psi}_L \Psi_R)$$

If $m = 0$ the two fields decouple and become independent. Thus they can be 2 different fields belonging to different representations.

Standard Model as a Chiral Theory

- 5 chiral fermion fields
- Quarks always belong to the fundamental $SU(3)_c$ representation
- Leptons are always singlet under $SU(3)_c$
- Left handed fields always below to the fundamental $SU(2)_L$ representation
- Right handed fields are always singlets under $SU(2)_L$
- There is not right handed neutrino field (it would be singlet under all simple subgroups)
- Having L,R particle in different representations means that the parity symmetry P is broken

F	$SU(3)$	$SU(2)$	$U(1)$
Q_L	3	2	1/6
u_R	3	1	2/3
d_R	3	1	$-1/3$
L_L	1	2	$-1/2$
e_R	1	1	-1

Table 1: Standard Model Fermion field content

$$P: x \to -x, t \to t, L \leftrightarrow R$$

Standard Model Fermion Lagrangian

The lagrangian is simply

$$\bar{\Psi} i D_\mu \gamma^\mu \Psi$$

Where Ψ is each of the 5 chiral fields, summed over all group indices, where D_μ needs to be written depending on the representation of the field Ψ. For example, for e_R, that is only charged under $U(1)_Y$,

$$D_\mu = \partial_\mu - i g_Y Q B_\mu$$

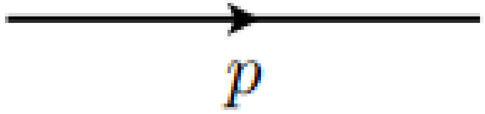
For L_L, that is charged under both $U(1)_Y$ and $SU(2)_L$,

$$D_\mu = \mathbb{I}\partial_\mu - i g_Y Q B_\mu \mathbb{I} - i g_L W_\mu^a t^a$$

Fermion Lagrangian Feynman rules

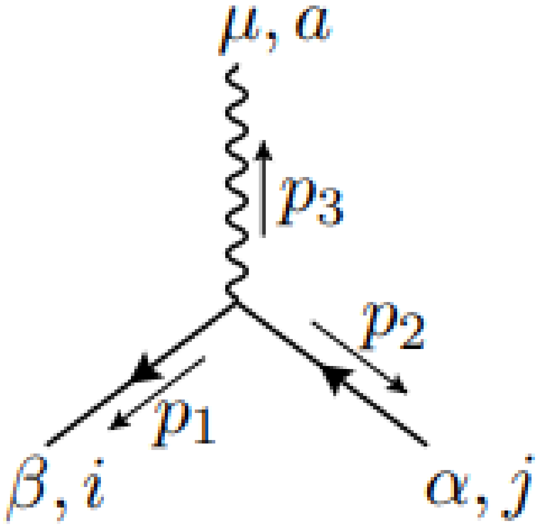
The term with only 2 Ψ fields gives, as usual, the propagator:

$$\frac{i(\gamma_\mu k^\mu + m)}{k^2 - m^2 + i\varepsilon}$$



The terms with 2 Ψ fields plus one gauge boson field gives fermion-gauge bosons interactions

$$ig\gamma^\mu t^a$$

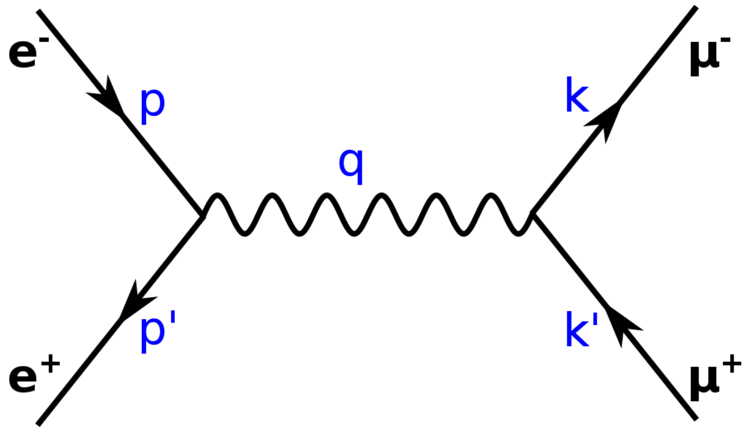


QED/QCD processes/exercises

Some examples

$e^+e^- \to \mu^+\mu^-$

- Write matrix element
- Get squared matrix element averaged over initial spins and summed over final spins
- Find the cross section in the c.o.m. frame



$e^+e^- \to e^+e^-$

- Write matrix element
- Get squared matrix element averaged over initial spins and summed over final spins
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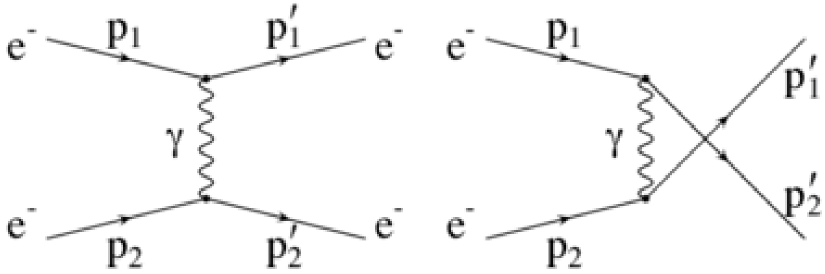
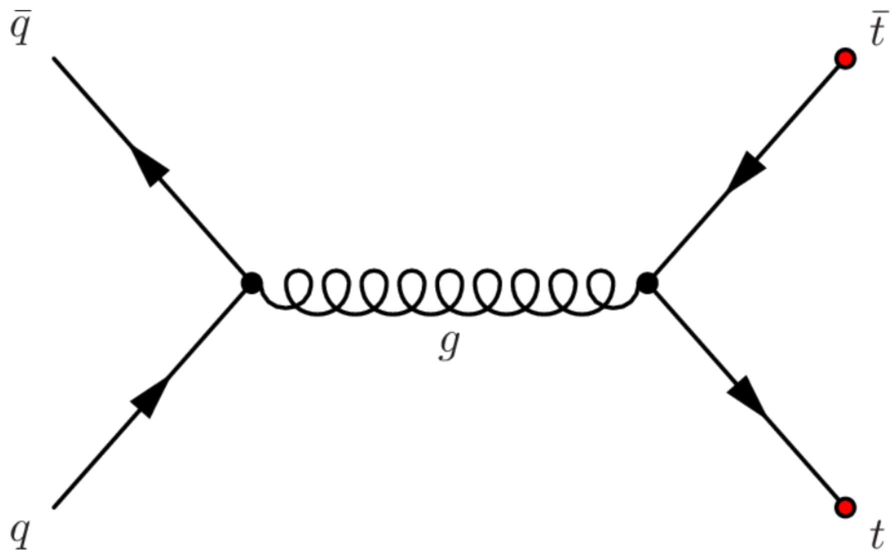


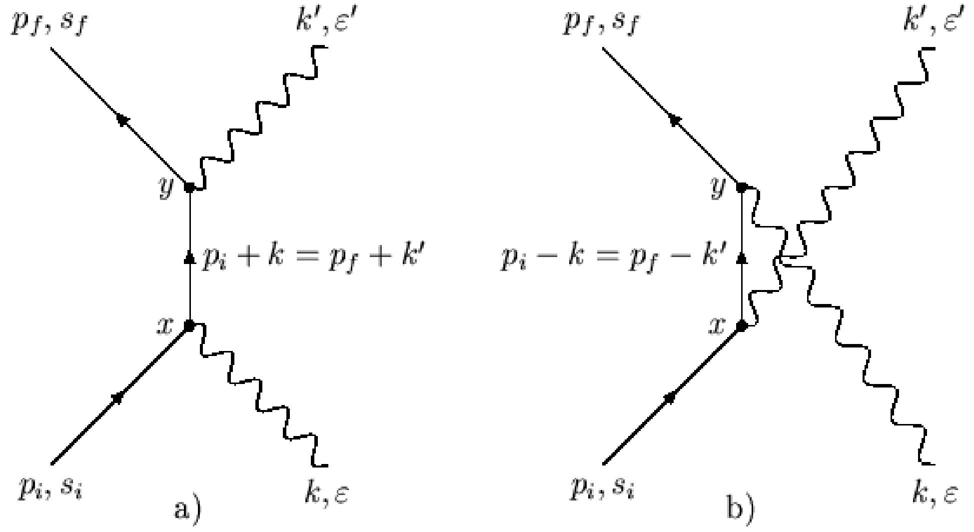
$q\bar{q} \to q'\bar{q}'$

- Write matrix element. Hint: can be obtained by analogy from QED
- Get squared matrix element averaged over initial spins/colors and summed over final spins/colors. Hint: can be obtained by analogy from QED
- Find the cross section in the c.o.m. frame



$e^+e^- \to \gamma\gamma$

- Write matrix element
- Check the ward identity for this process
- Get squared matrix element averaged over initial spins and summed over final polarizations
- Find the cross section in the c.o.m. frame



$q\bar{q} \rightarrow gg$

- Write matrix element. Note: there is an additional diagram comparing to QED
- Check the ward identity for this process