

Lecture 2: Particle dark matter

- What could DM be?
- Consider possibility DM represents 1+ new particle(s)
 - What is the DM mass?
 - How do we explain its observed abundance?

DM can be very heavy (e.g PBHs). For particles, up to Planck mass OK in principle.

How low in mass can we go?

- DM wavelength must be short enough to fit inside smallest observed DM structures
 - ~ 1 kpc ($\sim 3000 \text{ lyr} \sim 10^11 \text{ light-seconds} \sim 3 \times 10^{19} \text{ m}$)
 - 34 orders of magnitude larger than atomic nucleus, ~ 1 fm
 - 34 orders of magnitude lower energy than QCD scale, $\sim 10^{-22} \text{ eV}$
- More careful calculation + observations of small-scale structure
 - $\Rightarrow m_X \gtrsim 2-3 \times 10^{-21} \text{ eV}$ (note: we use X here & thereafter to denote DM)

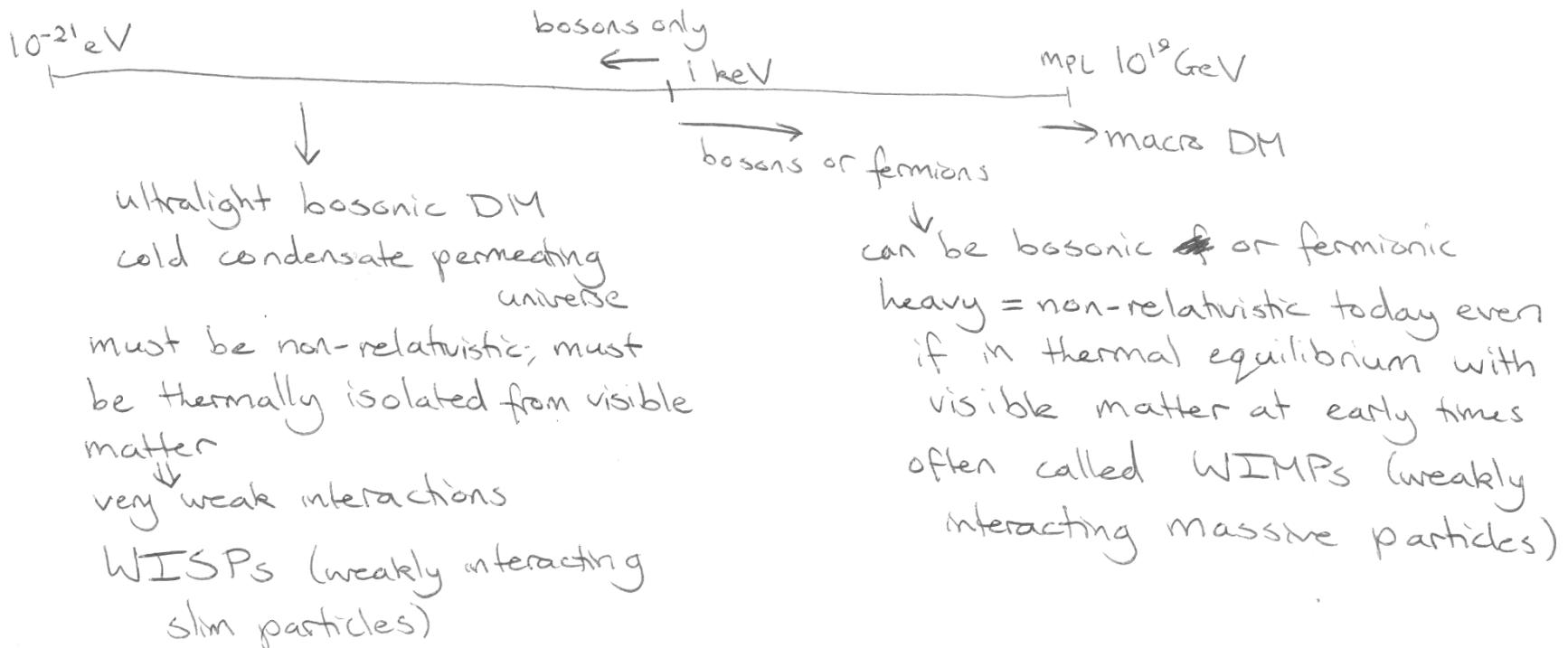
Enormous mass range!

For fermions we can do better - Tremaine-Gunn bound

$$\text{Phase space density} \sim \frac{n_X}{p^3} \sim \frac{\rho_X}{m_X (m_X v)^3} \lesssim 2 \text{ for fermions (Pauli exclusion)}$$

$$\Rightarrow m_X \gtrsim \left(\frac{\rho}{v^3}\right)^{1/4} \sim \left(\frac{1 \text{ GeV/cm}^3}{(10^{-5} \text{ s})^3}\right)^{1/4} \text{ for } \rho \sim 1 \text{ GeV/cm}^3, v \sim 10^{-5} \text{ s, typical small galaxy values}$$

$\sim \text{few hundred eV}$



Let us now consider implications if DM & visible matter are in thermal equilibrium in early universe.

Evolution of DM # density n_X

No interactions: $\frac{d}{dt}(n_X a^3) = 0 \Rightarrow a^3 \frac{dn}{dt} + 3a^2 n \frac{da}{dt} = 0 \Rightarrow \frac{dn}{dt} + 3Hn = 0$

i.e. $n_X \propto \rho_X \propto a^{-3}$ (for convenience write $n \equiv n_X$)

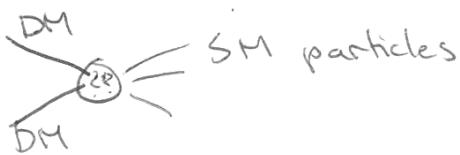
Now consider adding DM-SM (Standard Model) interactions to allow for equilibrium:

Options: (for # changing interactions)



decay

but we know DM is stable on lifetime of universe - decays should be v. rare in early universe



(2-body)
annihilation

analogous to particle-antiparticle annihilation



3-body

If DM is heavy, # density is small, rates for 3-body & higher processes are suppressed relative to 2-body (by extra powers of n_x)

So let us for now assume 2-body annihilation dominates # changing processes
(not necessarily true, e.g. if 2-body annihilation rate is zero, 3-body non-zero)

Rate controlled by $\langle \text{cov} \rangle$ thermally averaged rate coefficient
↓ cross-section

$$\frac{dn}{dt} + 3Hn = -\langle \text{cov} \rangle n^2 + \text{term from reverse reaction (DM produced from SM processes)}$$

↓ depletion by annihilation (rate is actually $\frac{1}{2} \langle \text{cov} \rangle n^2$, but each annihilation removes two DM particles)

→ independent of n , depends only on SM bath (e.g. on T & particle content)

$$\text{Write as } \frac{dn}{dt} + 3Hn = -\langle \text{cov} \rangle (n^2 - b(T))$$

Consider limit as $\langle \text{cov} \rangle \rightarrow \infty$ - drives n^2 toward $b(T)$

But increasing interaction rate should bring DM into thermal equilibrium with SM
i.e. drives n^2 toward $n_{\text{eq}}^2 \Rightarrow b(T) = n_{\text{eq}}(T)^2$

$$\Rightarrow \frac{dn}{dt} + 3Hn = -\langle \text{cov} \rangle (n^2 - n_{\text{eq}}^2)$$

n_{eq} is given approximately by the Boltzmann distribution

Ignoring $O(1)$ factors,

$$n_{\text{eq}} \sim \begin{cases} (m_X T)^{3/2} e^{-m_X/T}, & T \ll m_X \text{ (non-relativistic)} \\ T^3, & T \gg m_X \text{ (relativistic)} \end{cases}$$

When $\langle \text{cov} \rangle \rightarrow 0$, n evolves as $\frac{1}{a^3}$ (annihilations negligible compared to expansion)

When $\langle \text{cov} \rangle \rightarrow \infty$, $n \rightarrow n_{\text{eq}}$ (annihilations maintain equilibrium)

Crossover regime: $n^2 \langle \text{cov} \rangle \sim Hn$ (i.e. annihilation rate \sim expansion rate)

i.e. $n \langle \text{cov} \rangle \sim H$ - ~~we refer to this period as "freezeout"~~

Full calculation - solve evolution equation numerically

Now - simple estimate using scaling relations

First case: freezeout condition occurs while DM is relativistic

Before freezeout: $n \sim n_{\text{eq}} \sim T^3$

After freezeout: $n \propto a^{-3} \propto T^3$

$\Rightarrow n \sim T^3$ at all times

↳ temperature of photon bath

But also $n_g \sim T^3$ (blackbody distribution for photon bath)

and $n_g \sim 10^{10} \times \# \text{ density of baryons today}$

(photons have typical energy $\text{few} \times 10^4 \text{ eV}$
 $\sim \text{few} \times 10^{12}$ below proton mass

If DM has ~~#~~ density 10 o.o.m larger than
baryons, but only 5x mass density, implies
DM mass $\lesssim 10^{-9}$ baryon mass $\sim 1 \text{ eV}$

DM is neutrino-like - very light, decoupled
from SM while relativistic, gets its temperature
from interactions w/ SM

"hot dark matter" - like neutrinos, difficult to support structure formation

Could be a small fraction ($\leq 1\%$) of dark matter

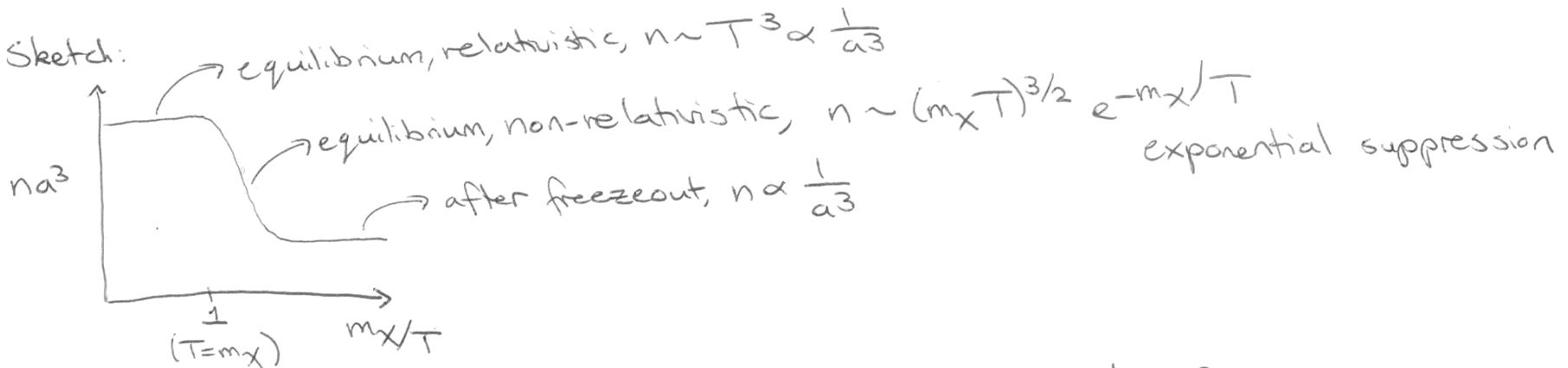
Basic issue: # density of relativistic species is *huge*

need a way to deplete it unless DM is $\lesssim 1 \text{ eV}$

one way to deplete it: keep DM in equilibrium while ^{it becomes non-}relativistic
equilibrium abundance exponentially suppressed

Case 2: freezeout occurs when DM is non-relativistic.

Sketch:



How big a drop do we need to get correct DM abundance?

By definition of freezeout

$$H_f \sim \langle \sigma v \rangle_{nf}$$

indicates "at freezeout"

During radiation domination
(valid for $m_X \gg 1\text{eV}$), $H \sim \frac{T^2}{MPL}$

$$\Rightarrow \frac{T_f^2}{MPL} \sim \langle \sigma v \rangle_{nf} \sim \langle \sigma v \rangle_{eq} T_{eq} m_X^2$$

$$\Rightarrow (\text{using } T_f \sim m_X) \langle \sigma v \rangle \sim \frac{1}{T_{eq} MPL}$$

Needed xsec is nearly independent of mass

Halving amount of DM would halve $T_{eq} \Rightarrow$
double $\langle \sigma v \rangle$ needed

i.e. needed $\langle \sigma v \rangle$ inversely proportional to amount of DM

↓
write in terms of temperature at matter-radiation equality (MRE)

At MRE, $p_{rad} \sim p_{DM}$

$$\Rightarrow n_{eq} m_X \sim T_{eq}^4$$

$$\begin{aligned} n_{eq} &= n_f \left(\frac{a_f}{a_{eq}} \right)^3 \text{ (not "in equilibrium"!)} \\ &\approx n_f \left(\frac{T_{eq}}{T_f} \right)^3 \end{aligned}$$

$$\Rightarrow n_f m_X \sim T_f^3 T_{eq}$$

Note $T_f \sim m_X$ since $\#$ is triggered by sharp density drop from DM going non-relativistic

$$\Rightarrow n_f \sim T_{eq} m_X^2$$

So far this is independent of particle physics (except for assuming 2-body annihilation - generalization to n-body annihilation is $\langle \text{cov}^{n-1} \rangle$ (rate coefficient) $\sim \frac{m_k}{m_{\text{Pl}} T_{\text{eq}}}^{4-2n}$)
 Can work for a wide range of DM masses.

~~Let's put in some numbers: $T_{\text{eq}} \sim 1 \text{ eV}$, $m_{\text{Pl}} \sim 10^{19} \text{ GeV} \sim 10^{28} \text{ eV}$~~

$$\Rightarrow \langle \text{cov} \rangle \sim \frac{1}{(10^{14} \text{ eV})^2} \sim \frac{1}{(100 \text{ TeV})^2} \xrightarrow{\text{some coupling}}$$

If we estimate xsec as $\langle \text{cov} \rangle \sim \frac{\alpha_D^2}{M^2}$ $\xrightarrow{\text{some mass scale associated w/ DM-SM interaction}}$ (e.g. for e^+e^- annihilation)
 then this suggests $M \sim \alpha_D (100 \text{ TeV})$

For weak coupling, $\alpha_D \lesssim 1$, $M \lesssim 100 \text{ TeV}$.

Careful calculation of unitarity bound (~~xsec is bounded above by unitarity for a given mass, for too-heavy DM cannot achieve this required xsec~~) finds $m_X \lesssim \text{few hundred TeV}$ (1407, 7874) for DM whose abundance is set by annihilation.

For electroweak-scale couplings, $\alpha_D \sim 10^{-2}$, suggests $M \sim 1 \text{ TeV}$ (or 100 GeV - I have made many approximations!)

Similar mass/coupling to known SM particles - does key to DM lie in electroweak physics? This observation is known as the "WIMP miracle".

(But remember this basic mechanism covers a much wider range of masses/couplings!)