Canberra International Physics Summer School
"Frontiers in Physics", ANU, 1-5 Dec 2014.

Mathematical Physics Quiz

Three Windows 8 tablet computers with pre-installed MS Office (valued $500 each) to be won in this competition.

The works have to be submitted in writing to cpss@anu.edu.au with the subject line "quiz". Due date 11:59pm Tuesday 2 December 2014 (as by "sent" date stamp).

Eligibility. The participant of the quiz

1. must be full time graduate or undergraduate university student from Australia or New Zealand as of semester 2 of 2014 (a full time undergraduate student means a student who should have completed at least 3 standard courses in the semester 2 of 2014).
2. must be registered and must attend the "Frontiers in Physics", ANU, 1-5 Dec 2014.

Problems

Problem 1

On the Summer School poster you can see a geometric object in the 3-dimensional Euclidean space $\mathbb{R}^3$. For brevity we will call it a *"space station"*. It contains 15 vertices, 28 edges, 18 planar quadrilateral faces, 18 circles and 4 spheres. Each vertex lies on at least one sphere. Each quadrilateral face is inscribed into a circle. Each circle lies on at least one sphere.

A. Explain how to construct the "space station" step by step, i.e., describe a detailed procedure starting with one point and then consequently adding all other points, edges, faces, circles and spheres. You are allowed to use basic geometry axioms and theorems

(i) Given any two different points, there is exactly one line containing them,

(ii) Given any three different noncollinear points, there is exactly one plane containing them,

(iii) Given any three different noncollinear points, there is exactly one circle containing them,

(iv) Given any four different noncoplanar points, there is exactly one sphere containing them,

(v) Any three planes, no two of which are parallel or coincide, intersect at one point only.

More complicated facts need to be proven or justified by a clear reference to the literature/textbooks.

B. How many lengths and angles one needs specify to uniquely determine the shape of the "space station" up to overall rotations and translations of the Euclidian space $\mathbb{R}^3$? For instance, to uniquely describe an arbitrary planar quadrilateral one needs to specify two of its sides and three angles.

C. The same question as **B** above, but should the "space station" be constructed in the 4-dimensional Euclidean space $\mathbb{R}^4$, instead of $\mathbb{R}^3$.

Problem 2

An electric charge $+Q$ is placed at a distance r from the centre of a hollow conductive sphere of a radius $R < r$. The sphere is "grounded" (i.e., its electric potential is equal to zero). Find the total induced electric charge of the sphere.

Problem 3

A thin lens is defined by the properties:

(i) Any light ray that passes through the center of the lens will not change its direction

(ii) Any light ray that enters parallel to the axis on one side of the lens proceeds towards the focal point F on the other side.

(iii) Any light ray that arrives at the lens after passing through the focal point on the front side, comes out parallel to the axis on the other side.

Figure 1 shows an object AB and its image CD in a thin lens. Find the position of the lens (its centre and orientation) and its focal distance. Justify your geometric construction.

Figure 1: Figure 1. The object (left) and its image (right) in a thin lens.

Problem 4

A relativistic spaceship is moving along the x-axis in the observer's inertial frame, such that at the time t its x-coordinate in this frame is equal to $x(t)$ (where $x(t)$ is some given function of t). Find the spaceship acceleration in its "momentarily comoving reference frame" (this is an inertial frame which at any given moment of time happens to be moving in the same direction and at the same speed, as the spaceship). This is the acceleration you would feel if traveling in this spaceship.

Problem 5

Three identical cylindrical logs are placed on a flat horizontal surface as shown in Figure 2. The sliding friction coefficient between the logs and between the surface and the logs is equal to μ. The Earth's gravity force $F = Mg$ is directed vertically downwards. Find the minimal value of the friction coefficient at which the logs do not slide.

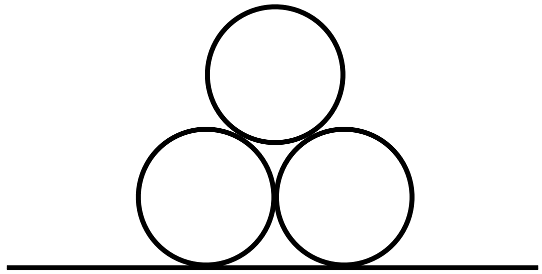

Figure 2

Problem 6

Two identical transformers with the winding turns ratio $2:1$ are connected as shown in Figure 3. Namely, the long coil of the first transformer is connected in series with the short coil of the second transformer to form an "input circit" marked by the letters A and B in the diagram. In addition, the short coil of the first transformer is connected in series with the long coil of the second transformer to form an "output circuit" marked by C and D. The input circuit is connected to a source of AC current with the voltage $V_{\text{input}} = 100\ Volts$. Find a voltage V_{output} between the points C and D of the output circuit.

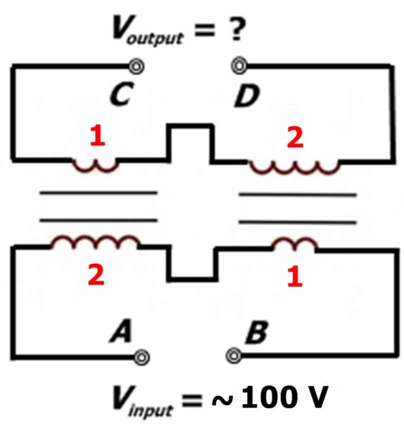

Figure 3

Assume that the transformers are ideal. An ideal transformer has perfectly conducting coils with a common magnetic flux, which is closed and completely confined inside the transformer. It implies that magnetic fluxes of two transformers are completely independent. The winding turns ratio $2:1$ means that one coil of the transformer has two times more turns than the other. These coils are marked by the numbers 2 and 1 and referred to in the text as the "long" and "short" coils, respectively.